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# INVESTIGATION OF THE CONSTANT OF SURFACE ROUGHNESS FUNCTION

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## ABSTRACT

The main objective of this paper is to throw some light on the constant of surface roughness function. Many investigators related that constant to the surface roughness density defined as the ratio of the total surface area to the roughness area. Others introduced alternative definition , to be the ratio of the total surface area to the total roughness frontal area normal to the flow .

The influence of pressure gradient is not considered by those investigators. Moreover, they suggested a unique value for this constant in the presence of pressure gradient .

In this research, the analytical study of the constant of surface roughness function regarding the pressure gradient in form of Euler number, shows variable values for different ratios of $k / \delta^{**}$.

## NOMENCLATURE

| | |
|---|---|
| A | velocity profile parameter |
| B | constant |
| $\bar{c}$ | velocity at the outer edge of the boundary layer (m/s) |
| $c_\tau$ | friction velocity , $\sqrt{\tau_w/\rho}$ ( m/s) |
| $c_\tau k/\nu$ | dimensionless roughness height |
| $c_\tau y/\nu$ | dimensionless distance normal to the wall |
| $c_x$ | velocity of the fluid inisde the boundary layer in x-direction (m/s |
| $C(\frac{c_\tau k}{\nu})$ | surface roughness function |
| $C_r$ | constant of surface roughness function |
| $H_{12}$ | boundary layer form parameter , $\delta^* / \delta^{**}$ |
| k | roughness height ( m ) |
| x | coordinate in the direction of the wall |
| y | coordinate normal to the direction of the wall |
| $\delta$ | boundary layer thickness ( m ) |
| $\delta^*$ | boundary layer displacement thickness, $\int_0^\infty (1 - \frac{c_x}{\bar{c}})\, dy$ ( m ) |
| $\delta^{**}$ | boundary layer momentum thickness, $\int_0^\infty \frac{c_x}{\bar{c}} ( 1 - \frac{c_x}{\bar{c}} )\, dy$ ( m ) |
| $\varkappa$ | von Kármán's universal constant |
| $\Lambda$ | Euler number , $- \frac{1}{\bar{c}} \cdot \frac{d\,\bar{c}}{d\,x} \cdot \delta^{**}$ |

| | |
|---|---|
| $\lambda$ | roughness density |
| $\nu$ | kinematic viscosity of fluid ( $m^2 / s$ ) |
| $\rho$ | density of fluid ( $kg / m^2$ ) |
| $\tau_w$ | wall shear stress ( $N / m^2$ ) |

## 1- INTRODUCTION

The investigators of previous studies indicated that the constant of surface roughness function, $C_r$, depends on the roughness geometry and density.

Bettermann [ 1 ] introduced the effect of roughness density , $\lambda$ , defined as the ratio of total surface area to roughness area, on the law of the wall. He has given the constant $C_r$ as function of the roughness density, $\lambda$ , for values of $1 \leqslant \lambda \leqslant 5$.

Dvorak [ 2 ] extended the correlation for $C_r$ to a wider range of roughness shapes and to roughness densities greater than five .

Simpson [ 3 ] defined the roughness density as the ratio of the total surface area to the total roughness frontal area normal to the flow. Since the form drag should be nearly proportional to the roughness frontal area this definition appears plausible physically. A generalization of the roughness density correlations of Dvorak and Bettermann has been found to be applied to the several roughness element geometries examined by Simpson .

This analytical study is based on Zancow [ 4 ] experimental work with rough surface and the velocity law of Rotta [ 5 ] .

## 2- BASIC EQUATIONS

### 2-1 VELOCITY DISTRIBUTION

Rotta [ 5 ] presented the rough surface law of the wall as :

$$\frac{c_x}{c_\tau} = \frac{1}{\varkappa} \left( \ln \frac{c_\tau y}{\nu} + 2A \frac{y}{\delta} \right) + B + C \left( \frac{c_\tau k}{\nu} \right) , \qquad (1)$$

in which $\varkappa$ and B are two empirical constants of o.4 and 5.2 values respectively .

The velocity at the outer edge of the boundary layer $\bar{c}$, i.e, at $y=\delta$ is deduced from equation (1) as follows :

$$\frac{\bar{c}}{c_\tau} = \frac{1}{\varkappa} \left( \ln \frac{c_\tau \delta}{\nu} + 2A \right) + B + C\left( \frac{c_\tau k}{\nu} \right) . \qquad (2)$$

From equations (1) and (2), the velocity distribution in the boundary layer may be obtained :

$$\frac{c_x}{\bar{c}} = 1 + \frac{c_\tau}{\bar{c}} \left[ \frac{1}{\varkappa} . \ln \frac{y}{\delta} - \frac{2\,A}{\varkappa} \left( 1 - \frac{y}{\delta} \right) \right] \qquad (3)$$

It may be noticed from the previous equation that the surface roughness (through $C\left(\frac{c_\tau k}{\nu}\right)$) has no effect on the velocity distribution .

## 2-2 SURFACE ROUGHNESS FUNCTION

The surface roughness function $C\left(\frac{c_\tau k}{\nu}\right)$ is zero for smooth surfaces, while for complete rough surface, Clauser [6] has shown that function takes the from :

$$C\left(\frac{c_\tau k}{\nu}\right) = C_r - \frac{1}{\varkappa} \cdot \ln \frac{c_\tau k}{\nu} \qquad (4)$$

with $C_r$ the constant of surface roughness function, which can be obtained by substituting $C\left(\frac{c_\tau k}{\nu}\right)$ from equation (4) into equation (2) :

$$C_r = \frac{\bar{c}}{c_\tau} + \frac{1}{\varkappa}\left(\ln \frac{k}{\delta} - 2A\right) - B \qquad (5)$$

The previous equation may be modified to include the ratio of roughness height to momentum thickness $k/\delta^{**}$, to become :

$$C_r = \frac{\bar{c}}{c_\tau} + \frac{1}{\varkappa}\left(\ln \frac{k}{\delta^{**}} + \ln \frac{\delta^{**}}{\delta^{*}} + \ln \frac{\delta^{*}}{\delta} - 2A\right) - B \qquad (6)$$

Here, the ratio $\delta^*/\delta^{**}$ is the form parameter, $H_{12}$ ; while $\delta^*/\delta$ is given by :

$$\frac{\delta^*}{\delta} = \frac{1}{\varkappa} \cdot \frac{c_\tau}{\bar{c}} \cdot (1 + A) \qquad (7)$$

From which the expression of the constant $C_r$ may be obtained as :

$$C_r = \frac{\bar{c}}{c_\tau} - B + \frac{1}{\varkappa} \ln \frac{1 + A}{\varkappa} - \frac{2A}{\varkappa} + \frac{1}{\varkappa} \ln\left(\frac{k}{\delta^{**}} \cdot \frac{1}{H_{12}} \cdot \frac{c_\tau}{\bar{c}}\right) \qquad (8)$$

where $c_\tau / \bar{c} = \left(\tau_w / \rho \bar{c}^2\right)^{1/2}$

A computer program has been developed to get the constant $C_r$ using Zancow [4] experimental results for flow over rough surfaces .

The constant $C_r$ given by equation (8) can be represented by $C_r = C_r\left(A, \frac{k}{\delta^{**}}, \frac{c_\tau}{\bar{c}}, H_{12}\right)$ which may be simplified by making use of the relation $A = A\left(\Lambda, \frac{k}{\delta^{**}}\right)$ ; illustrated in Fig . (1) ; $H_{12} = H_{12}\left(\Lambda, \frac{k}{\delta^{**}}\right)$ and $\frac{\tau_w}{\rho \bar{c}^2} = \frac{\tau_w}{\rho \bar{c}^2}\left(\Lambda, \frac{k}{\delta^{**}}\right)$ ; according to Zancow [4] ; in the from :

$$C_r = C_r\left(\frac{k}{\delta^{**}}, \Lambda\right) \qquad (9)$$

that relation is represented in Fig . ( 2 ) .

## 3 - DISCUSSION OF RESULTS

The influence of pressure gradient in from of Euler number, $\Lambda$, on the constant $C_r$ can be deduced from Fig. (2). Flows with zero pressure gradient are corresponding to $\Lambda = 0$, while with adverse pressure gradient is given by $\Lambda > 0$ .

Figure (2) represents the variation of the constant $C_r$ with the ratio $k/\delta^{**}$ and Euler number, $\Lambda$, as a parameter. The chart contains four curves for boundary layers at

$\Lambda = 0, 1.10^{-3}, 2.10^{-3}$ and $3.10^{-3}$ .

For constant Euler number ; $\Lambda$ = const. ; each curve can be divided into two parts, the first part for values of the ratio $k/\delta^{**} < 0.1$ , at which the constant $C_r$ decreases as the ratio $k / \delta^{**}$ increases. In the second part ; $k / \delta^{**} > 0.1$ ; the constant $C_r$ increases with the increase of $k / \delta^{**}$ .

For the same values of the ratio $k / \delta^{**}$ , the constant $C_r$ increases as Euler number , $\Lambda$ , increases. Also, at ratios of $k / \delta^{**} < 0.1$, the decreasing rate of $C_r$ increases with an increment in Euler number .

## CONCLUSIONS

As a result of the present investigation the following conclusions are obtained: For certain ratio of $k / \delta^{**}$ ; $k / \delta^{**} = 0.1$ ; the constant $C_r$ is a minimum for all Euler numbers and as this ratio is increased or decreased , $C_r$ increases. A similar observation was made by Bettermann [1] and Liu et al. [7] for the variation of $C_r$ with the roughness density ,$\lambda$.

The results obtained for the constant $C_r$ show a variation in its value with the pressure gradient in form of Euler number for different ratios of $k/ \delta^{**}$. This contradicts the suggestion of Rotta [5] and others to remain the constant $C_r$ unaffected by the existence of pressure gradient.

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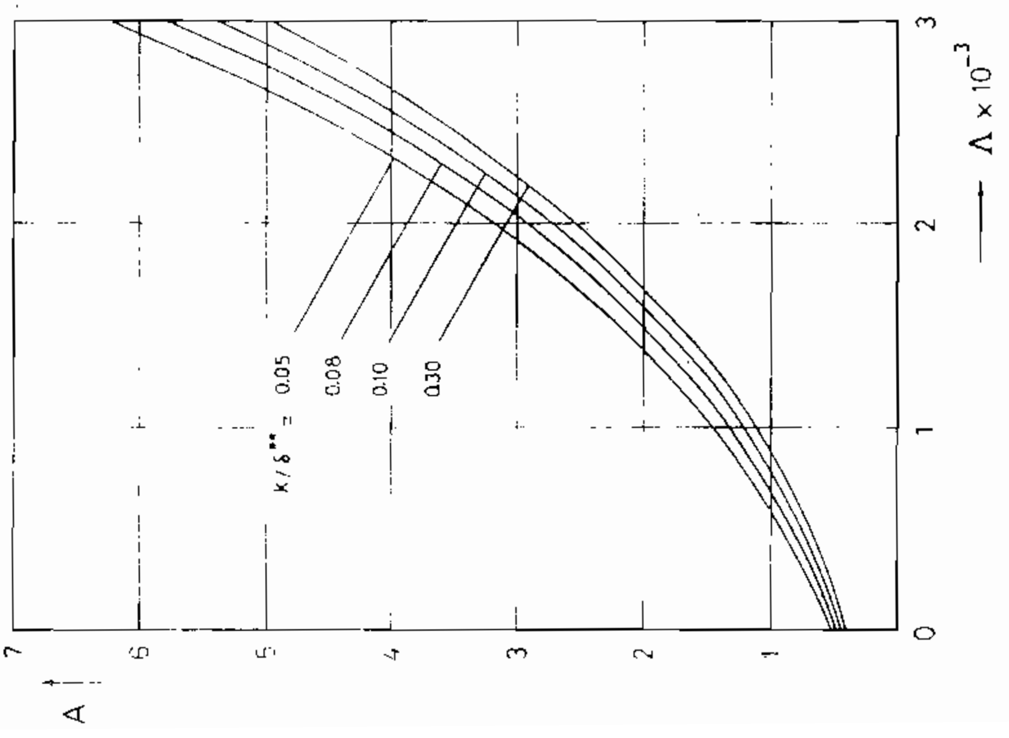

Fig.(1) Velocity profile parameter versus Euler number with the ratio k/δ** as parameter

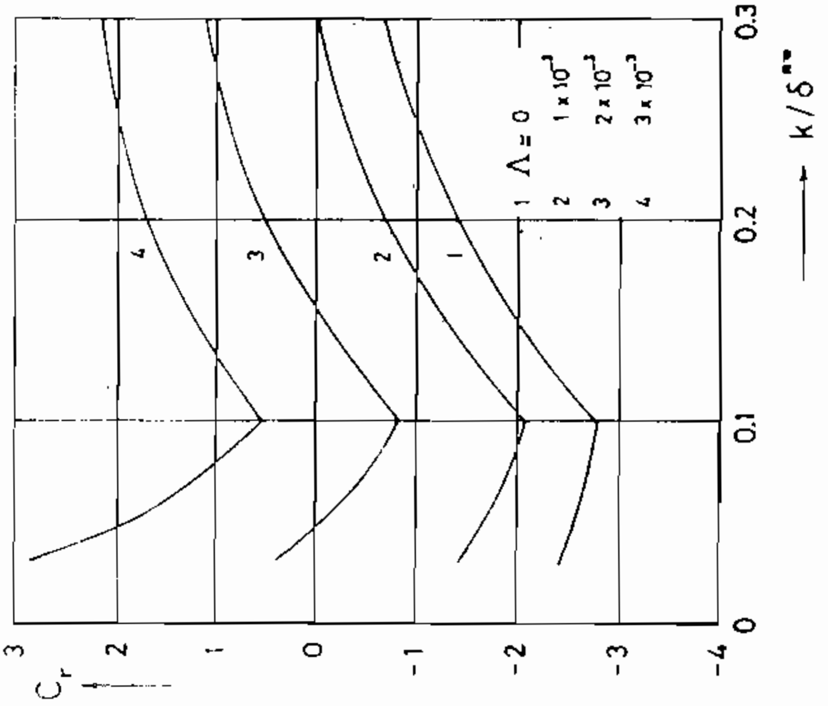

Fig.(2) Constant of surface roughness function versus the ratio k/δ** with Euler number as parameter