Menofia University
Faculty of Engineering Shebien El-kom
Basic Engineering Sci. Department.
Academic Year : 2017-2018
Date : 28 / 05 / 2018

Subject: Numerical Analysis (1)
Code: BES601
Time Allowed: 3 hours
Year: Master (Grade 600)
Total Marks: 100 Marks

Answer all the following questions: [100 Marks]

Q.1 (A) **By using *Differential Transform Method* (DTM) to solve the following simultaneous differential equations:** [20]

(i) $2\frac{dx}{dt}+\frac{dy}{dt}-x-y=e^{-t}$, $\frac{dx}{dt}+\frac{dy}{dt}+2x+y=e^{-t}$

with initial conditions: $x(0)=2, y(0)=1$

(ii) $\frac{dx}{dt}+\frac{1}{2}\frac{dy}{dt}+x=1$, $\frac{1}{2}\frac{dx}{dt}+\frac{dy}{dt}+y=0$

with initial conditions: $x(0)=0,\ y(0)=0$

(iii) $\frac{dx}{dt}+\frac{dy}{dt}+x+y=1$, $\frac{dx}{dt}=2x+y$

with initial conditions: $x(0)=0, y(0)=1$

(iv) $\frac{dx}{dt}=z-\cos(t)$, $\frac{dy}{dt}=z-e^{t}$, $\frac{dz}{dt}=x-y$

with initial conditions: $x(0)=0, y(0)=0, z(0)=2$

(B) **Using *Differential Transform Method* (DTM) to solve the differential equation:**

(i) $\frac{d^2v}{dt^2}-2\frac{dv}{dt}-8v=0$

with initial condition: $v(0)=3, v(0)=6$

(ii) $\frac{d^2v}{dt^2}+7\frac{dv}{dt}+10v=4e^{-3t}$

with initial conditions: $v(0)=0, v(0)=-1$

(iii) $u'''(t)=-u.$

with initial conditions: $u(0)=1, u'(0)=-1, u''(0)=1$

(iv) $u'''(t)=e^{t}, 0\le t\le 1,$

Subject to the boundary conditions: $u(0)=3, u'(0)=1, u''(0)=5.$

Then show that the exact solution is $u(t)=2+2t^2+e^{t}$

(C) **Using *Differential Transform Method* (DTM) to solve the following linear system of non-homogeneous differential equations**

$y'_1(x)=y_3(x)-\cos(x)$, $y'_2(x)=y_3(x)-e^{x}$ and $y'_3(x)=y_1(x)-y_2(x)$

with initial conditions: $y_1(0)=1, y_2(0)=0,$ and $y_3(0)=2$

Q.2 **(A) Using *the Adomian Decomposition Method* (ADM) to solve the following system of differential equations**

$$y_1' = y_3 - \cos(x), \quad y_2' = y_3 - e^x \quad and \quad y_3' = y_1 - y_2$$

with initial conditions: $y_1(0) = 1, y_2(0) = 0,$ and $y_3(0) = 2$

(B) Using *the Adomian Decomposition Method* (ADM) to solve the following non-linear ordinary differential equations:

(i) $y' - y^2 = 1$

with the initial conditions: $y(0) = 0$

(ii) $y''' = \frac{1}{x} y + y'$

with the initial conditions: $y(0) = 0, \; y'(0) = 1 \; and \; y''(0) = 2.$

Then show that the exact solution is: $y(x) = xe^x$

(C) The governing equation of a uniform Bernoulli–Euler beam under pure bending resting on fluid layer under axial force is:

$$EI \frac{\partial^4 v}{\partial x^4} + P \frac{\partial^2 u}{\partial y x^2} + K_f \, v + F(x,t) = 0, \quad 0 \le x \le L_e.$$

with boundary conditions (Clamped-Simply supported):

at $x = 0, \quad W(x) = \frac{dW(x)}{dx} = 0$

at $x = L_e, \quad W(x) = \frac{d^2W(x)}{dx^2} = 0$

Solve the beam equation problem using **the *Adomian Decomposition Method*** (ADM). Then compared the results with exact solutions. in the following form:

$$P = K_f = 0, \; F(x,t) = 1$$

Q.3 **(A) Solve using the *Homotopy perturbation method* the nonlinear system of equations**

$$u_t = uu_x + vu_y, \qquad v_t = uv_x + vv_y$$

with the initial condition: $u(x,y,0) = v(x,y,0) = x + y$

(B) Consider the following three-dimensional Helmholtz equation in the following form:

$$a \frac{\partial^2 u}{\partial x^2} + b \frac{\partial^2 u}{\partial y^2} + \lambda \, u = F(x,y)$$

with initial conditions: $u(0, y) = f_1(y), \quad u_x(0, y) = f_2(y),$

$$u(x, 0) = f_3(x), \quad u_y(x, 0) = f_4(x)$$

where $F(x, y)$, $f_1(y)$, $f_2(y)$, $f_3(x)$, $f_4(x)$ and a, b, λ are given functions and given constant respectively.

Solve the two-dimensional Schrodinger equation using the *differential transform method* (*DTM*), in the following form:

$$F(x, y) = (12\, x^2 - 3\, x^4) \sin(y)$$

$$a = b = 1, \lambda = -2 \text{ and } f_1(y) = 0, \; f_2(y) = 0$$

Q.4 **(A) Using *the Homotopy perturbation method* (HPM) to solve the following non-homogeneous one-dimensional unsteady heat problem:** [20]

(i) $u_t = u_{xx} + \sin x$

Subjected to the initial condition: $u(x, 0) = \cos x$.

Then compare your solution with the exact solution:

$$u(x, t) = \cos x \; e^{-t} + \sin x \,(1 - e^{-t})$$

(ii) $u_t + u\, u_x = x$

Subjected to the initial condition: $u(x, 0) = 2$

Then compare your solution with the exact solution:

$$u(x, t) = 2\, \mathrm{sech}\, t + x \tanh t$$

(iii) $u_{tt} = -u_{xxxx}$

Subjected to the initial condition: $u(x, 0) = \sin \pi x + 0.5 \sin 3\pi x$

(iv) $\frac{\partial u}{\partial t}(x, t) + i \frac{\partial^2 u}{\partial x^2}(x, t) = 0$

with the indicated initial condition: $u(x, 0) = e^{3ix}, \; x \in R$

(B) Consider the nonlinear singular initial value problem:

$$y'' + \frac{2}{x} y' + 4(2\, e^{y} + e^{y/2}) = 0$$

with initial conditions: $y(0) = 0, \quad y'(0) = 0,$

Solve the nonlinear singular initial value problem using the *adomian decomposition method* (ADM).

Q.5	**(A) Solve the following nonlinear Schrodinger equation with the indicated initial condition by applying the *Homotopy perturbation method:*** $$i\,u_t + u_{xx} + 2\,\lvert u\rvert^2\,u = 0$$ $$u(x,0) = e^{ix}$$ **(B) Consider the following Riccati equation:** $$y'(t) = -(3 - y(t))^2,$$ with initial conditions: $y(0) = 1$ Solve the Riccati equation using the *adomian decomposition method* (ADM).	[20]

Good Luck

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