Menofia University **Faculty of Engineering** **Shebien El-kom** **First Semester Examination** **Academic Year : 2012-2013**		**Department : Electrical Eng.** **Year : 3rd** **Subject : Eng. Mathematics** **Time Allowed : 3 hours** **Date : 16/1/2013**

Allowed Tables and Charts : None

Answer all the following questions: [70 Marks]

Q.1

(A) Show that the set of functions $\{sin(n\pi x)\}$, $n = 1, 2, 3, \ldots$ are orthogonal on the interval (0, 1). **[3 Marks]**

(B) Expand the function $f(x) = x$ in terms of $sin(n\pi x)$, on the interval (0,1) using the generalized Fourier series. **[3 Marks]**

(C) Solve the P.D.E. $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + (\sin 5x)e^{-2t}$, $0 < x < \pi$, $t > 0$.

Subject to the boundary conditions: $u(0,t) = 0$, $u(\pi,t) = 1$

and initial condition $u(x,0) = 0$ **[8 Marks]**

(D) Consider the following PDE

$$\frac{\partial^2 u}{\partial x^2} = -2\frac{\partial u}{\partial t}, \qquad 0 < x < 1, \qquad t > 0$$

Subject to: $u(x,0) = 0$, $u(0,t) = 0$, *and* $\frac{\partial u(1,t)}{\partial x} = 1$

State the type of the equation (parabolic, elliptic or hyperbolic) and the boundary conditions (Dirchlet, Neumman, Robin or mixed) then solve the equation for $u(x,t)$.

[8 Marks]

(E) Express each equation in terms of conjugate coordinates:

(i) $2x + y = 5$ **(ii)** $x^2 + y^2 = 36$ **[4 Marks]**

(F) *i)* Prove that the function $u = 2x(1 - y)$ is harmonic.

ii) Find a function v such that $f(z) = u + i v$ is analytic.

iii) Express $f(z)$ in terms of z. **[12 Marks]**

(G) Find the cube root of 10 using Newton-Raphson method, take $x_o = 2.1$.

[6 Marks]

Q.2

(A) (i) find the real root of the equation $x^3 - x - 11 = 0$ in the interval $2 < x < 3$ using the method of bisection.

(ii) Discuss (ناقش) with graphs three drawbacks (عيوب) of Newton-Raphson method for solving non linear algebraic equation. **[4 Marks]**

(B) (i) Solve the following equation $x e^x - 2 = 0$, using Newton-Raphson method. Take $x_o = 0$, make two iterations.

(ii) Use the formula $\cosh(x) \approx 1 + \frac{x^2}{2!} + \frac{x^4}{4!}$ to evaluate $\cosh(x)$ and find the truncation error.

[4 Marks]

(C) Use Euler's and Rung-Kutta 4th order method to solve the differential equation $\frac{dy}{dx} = y - x^2 + 1$ to obtain the value of y at $x = 0.4$ knowing that $y(0) = 0.5$ (take $h=0.4$).

[6 Marks]

(D) Given the following linear system of algebraic equations:

$$\begin{aligned} x_1 + 4x_2 + 2x_3 &= 15 \\ 2x_1 + x_2 + 5x_3 &= 19 \\ 3x_1 + x_2 + x_3 &= 8 \end{aligned}$$

(i) If you solve this system without ordering the equations, What do you expect? Discuss the convergence of this system through Scarbora criteria.

(ii) Order your equations in an appropriate way. Use Gauss-Seidel iterative method to make **two iterations**.

Use $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 1$. **[6 Marks]**

(E) Show that the function $w = f(z) = z^2 + 3z$ is analytic. Then evaluate $\int_C f(z)dz$ along

(i) The straight line from (2,0) to (0,2)

(ii) The straight line from (2,0) to (2,2) and then to (0,2).

[6 Marks]

With my best wishes

Dr. Islam M. Eldesoky