Menoufia University Faculty of Engineering, Shebin El-Kom, Basic Engineering science Department Second Semester Examination, 2014-2015 Date of Exam: 1/6/2015	MENOUFIYA UNIVERSITY	Subject: Ordinary Differential Equations (1). Code : BES 609 Year : postgraduate students Time Allowed : 3 hours Total Marks: 100 marks

Answer the following questions

Question 1 **(50 marks)**

a) Use the method of Frobenius to find one solution near $x=0$ of the equation

$$x^2y''-xy'+y=0$$

b) Consider the nonlinear system

$$x'=-x+3y^2, \qquad y'=-y, \qquad z'=3y^2+z$$

Solve this system and compute the stable and unstable for the equilibrium at the origin.

c) write the following system in matrix form:

$$y_1'=y_1+2y_2+2e^{4t}$$
$$y_2'=2y_1+y_2+e^{4t}$$

- Conclude that every initial value problem for above system has a unique solution on $(-\infty,\infty)$.
- Verify that

$$\mathrm{y}=\frac{1}{5}\begin{bmatrix}8\\7\end{bmatrix}e^{4t}+c_1\begin{bmatrix}1\\1\end{bmatrix}e^{3t}+c_2\begin{bmatrix}1\\-1\end{bmatrix}e^{-t}$$

is a solution of the above system for all values of the constants c_1 and c_2.

- Find the solution of the initial value problem

$$\mathrm{y}'=\begin{bmatrix}1&2\\2&1\end{bmatrix}\mathrm{y}+\begin{bmatrix}2\\1\end{bmatrix}e^{4t},\ \mathrm{y}(0)=\frac{1}{5}\begin{bmatrix}3\\22\end{bmatrix}$$

Question 2 **(50 marks)**

a) Write briefly the steps of the Runge-Kutta algorithm to solve the differential equations.

b) Solve the initial value problem using Runge-Kutta Method with step size h=0.2 on the interval [1,2] , then compare the approximate solution with the actual solution :

$$x^2 y'' - xy' + y = 0, \qquad y'(1) = 2,\ y(1) = 4.$$

c) Find $y(1)$ for $y' = y - x;\ y(0) = 2$. using Euler's method with h=0.25.

d) A cylindrical tank is receiving and discharge water at the same time. Initially the tank is empty and at time t the depth is h. h and t are replaced by the equation

$$\frac{dh}{dt} + kh = kh_0\, e^{-kt}$$

- Find the depth of water as a function of t and sketch the graph of h against t.
- Sketch the direction field.

With my best wishes *Dr. Eng. Rizk Masoud*