

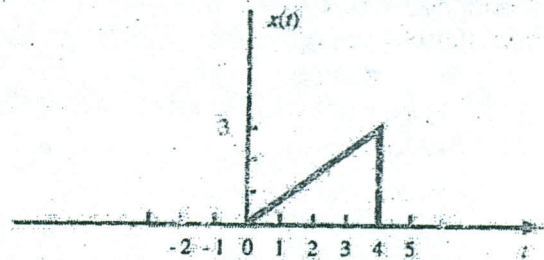


Attempt all questions. Assume any missed data. Full mark is 100

Q.1.a) A continuous-time signal $x(t)$ is shown in figure. Sketch and label each of the following signals:

- $x(t-2)$
- $x(2t)$
- $x(t/2)$
- $x(-t)$
- $x(n)$ if $T=1$

[5 Marks]



Q.1.b) Determine whether the signal $x(n) = 2 + \sin \frac{\pi n}{3} + \cos \frac{\pi n}{4}$ is periodic or not. If it is periodic, find its fundamental period. Is it an energy signal or a power signal? [5 Marks]

Q.1.c) Evaluate each of the following integrals:

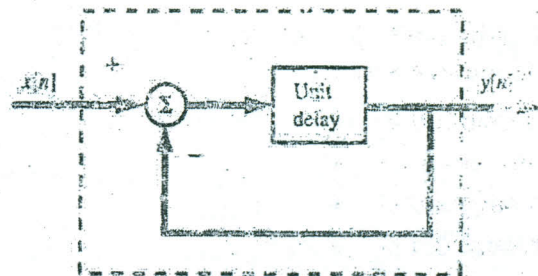
- $\int_{-\infty}^0 e^{-at} \delta(t-1) dt$
- $\int_{-2}^2 2^t \delta(2t-2) dt$
- $\int_{-\infty}^{\infty} \sin(2t) \delta''(t) dt$

[5 Marks]

Q.1.d) Find the input-output relation of the feedback system shown.

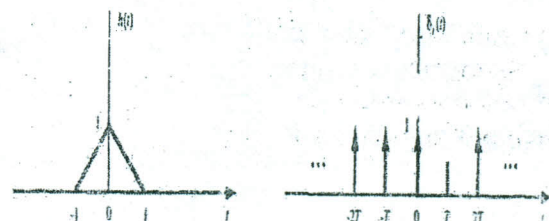
- Is the difference equation recursive?
- Find the Impulse response of the system
- Is the system FIR or IIR?
- Is the system causal?
- Is the system stable?

[5 Marks]



Q.1.e) The input $x(t)$ to an LTI system and the impulse response of the system are shown in the corresponding figure. Determine and sketch the output of the system when $T=1.5$

[5 Marks]



Q.2.a) For the transfer function $H(s) = \frac{se^{-3s}}{s^2 + 2.25s + 0.5}$

- Sketch the pole-zero plot for this transfer function.
- What are the possible ROC's for this transfer function?
- For each ROC in (ii), determine stability and causality of the system.
- For each ROC in (ii), determine the associated impulse response.

[8 Marks]

Q.2.b) An LTI system has a unit step response given by $(1 - e^{-t})u(t)$. For a certain unknown input, $x(t)$, the output is observed to be $(2 - 3e^{-t} + e^{-2t})u(t)$. For this observed output, determine the true input to the system as a function of time.

[5 Marks]

Q.2.c) Consider an LTI system described by the differential equation

$$y''(t) - y'(t) - 6y(t) = x(t), \quad y(0) = 1, \quad y'(0) = 0$$

- Find the system function. Locate poles and zeros in the s-plane.
- Find the impulse response of the system.
- Find the output of the system if $x(t) = e^t u(t)$.
- What are the zero-input response and the zero-state response?

[12 Marks]

Q.3.a) A system has the transfer function

$$H(z) = \frac{5}{1 - 0.7e^{j\pi/4}z^{-1}} + \frac{1}{1 - 0.7e^{-j\pi/4}z^{-1}} + \frac{2}{1 + 3z^{-1}}$$

Find the impulse response assuming the system is:

- stable
- causal
- Can this system be both stable and causal?

[5 Marks]

Q.3.b) Find the inverse Z-transform of each of the following functions:

i. $X(z) = \frac{z}{(z-1)(z-2)} \quad 1 < |z| < 2$

ii. $X(z) = e^{a/z} \quad |z| > 0$

[8 Marks]

Q.3.c) Consider a system described by

$$y(n) - 3y(n-1) = x(n), \quad y(-1) = 1, \quad x(n) = 4u(n)$$

- Find the system function and locate its poles and zeros in the complex plane.
- Find the impulse response.
- Determine the output of the system.
- Express the output $y(n)$ as a sum of two components; the zero-state response and the zero-input response.

[12 Marks]

Q.4.a) Consider a periodic square wave $x(t)$ given by:

$$x(t) = \begin{cases} 8 & 0 \leq t \leq 1 \\ 0 & 1 \leq t \leq 2 \end{cases}, \quad x(t) = x(t+2)$$

- i. Find the trigonometric Fourier series of $x(t)$
- ii. If $x(t)$ is applied as an input to a low-pass filter with frequency response

$$H(\omega) = \begin{cases} 1 & |\omega| \leq 4\pi \\ 0 & |\omega| > 4\pi \end{cases}$$

Find the output of the filter.

[7 Marks]

Q.4.b) Consider an ideal low-pass filter given by:

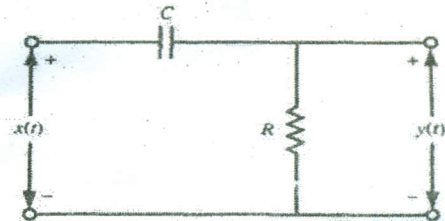
$$H(\omega) = \begin{cases} 1 & \omega \leq \omega_c \\ 0 & \omega > \omega_c \end{cases}$$

The input to this filter is $x(t) = \frac{\sin(at)}{\pi t}$. Find the output $y(t)$ for $a < \omega_c$. Repeat for $a > \omega_c$.

Comment on results.

[5 Marks]

Q.4.c) Derive an expression for the frequency response of the shown circuit. Sketch the magnitude and phase of the frequency response. Indicate the cut-off frequency on your sketch. Choose suitable values for R and C to realize a cut-off frequency of 10KHz.



[5 Marks]

Q.4.d) Sketch the Bode plot for the following frequency response

$$H(\omega) = \frac{100(1 + j\omega)}{(10 - j\omega/100)(1 + j\omega/10)}$$

[8 Marks]

My best wishes to all of you!

Assis. Prof. Hossam El-Din Moustafa