

## CALCULATION OF SPINNING RESERVE OF THE EGYPTIAN UNIFIED POWER SYSTEM

### حساب الإحتياطي الدائر للشبكة المصرية الموحدة

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#### الخلاصة:

ترجع أهمية هذا البحث إلى تقدير العول للشبكة الكهربائية الموحدة بجمهورية مصر العربية والتعرف على مشاكل تشغيل هذه الشبكة ومدى إتاحة وحدات التوليد المختلفة وحدود التغيير في الجهود والتذبذبة عند وقوع حوادث في الشبكة ومعدلات فشل مكونات محطات التوليد المختلفة كخطوة لتجميع هذه البيانات في قاعدة معلومات واحدة.

ويتناول هذا البحث حساب الإحتياطي الدائر وإحتمالات الخروج الإضطرابي لوحدات التوليد المختلفة بالشبكة الكهربائية الموحدة، وإعتمادا على معدلات الخروج المحسوبة لهذه الوحدات خلال فترة الدراسة وتقدير الإحتياطي الدائري بإستخدام عدد من الطرق المعروفة أهمها "طريقة ذات الحدين" و "طريقة بواسون" و "طريقة مونت كارلو" والتي تستخدم لتحديد الإحتياطي اللازم على المدى القصير والبعيد. وتم إستخدام البيانات الإحصائية لوحدات التوليد المختلفة في فترة زمنية معينة وذلك لحساب معدل الخروج الإضطرابي لوحدات التوليد.

وقد تم حساب الإحتياطي الإستاتيكي (على المدى السعيد) اللازم لضمان مستوى إعتيادية معينة عند مستويات مختلفة لتقدير الحمل في الشبكة الموحدة وتم مقارنة النتائج بأستخدام مجموعتين مختلفتين لمعدل الخروج الإضطرابي لوحدات التوليد حيث أن المجموعة الأولى تشمل القيم الحقيقية المحسوبة من واقع هذه الدراسة والمجموعة الثانية تشمل القيم المتفق عليها عالميا.

24% من حالات الخروج الإضطرابي للوحدات البخارية بسبب مشاكل التيزان وكان متوسط عدد ساعات الخروج بسبب مشاكل التيزان مرتفعة بصفة عامة عن مشاكل باقي النظم ما عدا التربينه فكان مساويا لها، كذلك تسببت مشاكل المكثف في خروج وحدات التوليد البخارية في 15.5% من مجموع حالات الخروج الإضطرابي الكلية مما يحتاج إلى إعادة دراسة لمشاكل كل من التيزان والمكثف لمحاولة تخفيض عدد مرات وساعات الخروج.

تسببت مشاكل الشبكة الكهربائية الموحدة في خروج وحدات التوليد البخارية في 8.7% من مجموع حالات الخروج الكلية مقابل 11% من مجموع حالات الخروج الإضطرابي للوحدات الغازية. لم يكن هناك أخطاء بشرية كمسببات للخروج الإضطرابي، ومن نتائج هذا البحث على الشبكة الكهربائية الموحدة الخاصة بوحدات التوليد يتضح أن مؤشرات الأداء للشبكة المصرية في نفس المستوى للشبكات العالمية المستقر.

#### Abstract:

The importance of this paper is to evaluate the reliability of the unified power system arising from the necessity of the recognition of the network problems, the availability of the generating units and the voltage and frequency variation limits in case of network accidents and the failure rates of the power plants components as a step towards the collection of this data.

The probability of forced outage of the different generating units and the spinning reserve of the unified power system are calculated by three methods: the binomial distribution method, the poisson method and monte carlo method which are usually used to determine both short and long term operating reserve requirement.

For computing the forced outage rate of the different generating stations, the statistical data of the different units has been used. Computer programs were designed compiled and applied.

40% of the forced outage events of the steam units are due to boiler problems, the average outage hours due to boiler problems is in general higher than those of the other system problems except the turbine problems which are equal.

The condenser problems causes 15.5% of the total forced outage events of the steam units which requires further study of the boiler and condenser problems in order to reduce the number of outage events and the average outage hours. The unified power system problems causes 8.7% of the total outage events of the steam units and 11% of the total forced outage events of the gas turbine units.

### 1. Introduction:

In this paper, the probability of forced outage of the different generating units and the spinning reserve of the Egyptian unified power system [1-3] was calculated by three methods: the binomial distribution method, the poisson method and the monte carlo method. These methods are usually used to determine both short and long term operating reserve requirement. For computing the forced outage rate of the different generating stations, the statistical data of the different units has been used. Computer programs were designed, compiled and applied.

### 2. Spinning Reserve Requirements:

At all times it is required that a power system operates a number of generating units whose capacity exceeds the actual demand to fulfil both the short and long term reserve requirements.

#### 2.1 Short term reserve:

The short term reserve or the operating reserve may be defined as that reserve above system load required for regulation within the hour to cover the minute to minute variations, the load forecast error and the unscheduled loss of generating units.

#### 2.2 Long term reserve:

Long term reserve is the static reserve required to secure a predetermined reliability level [4-5] which will be hopefully satisfied in the long period. A basic difference exists between the generating units probabilistic statistical data used in a static reserve study and that used in an operating reserve study. In case of a static reserve study, the basic statistics required are the generating units forced outage rate. In case of an operating reserve study, the basic statistics required are the probability that a generating unit will be removed from service due to a forced outage and will not be replaced by another unit.

In the static reserve study, the available generating capacity under normal conditions is the entire system installed capacity. In the operating reserve study, the assumption used is that, there is an excess installed capacity in the system such that if a unit is forced out of service it is only a matter of time to replace it by another unit to meet the load requirement. In this paper static and operating reserve requirements are discussed and evaluated.

### 3. Binomial Distribution Method:

Consider the forced outage rate (P) for each unit of a system of (n) units of equal size. The probability (q) that an outage does not occur will be:

$$q = 1 - P$$

if the probability of one unit outage is (P), then the probability of two units outage is ( $p^2$ ) and the probability of (r) units outage is ( $p^r$ ). The probability that (n-r) units outage does not occur is:

$$q^{n-r} = (1-p)^{n-r}$$

Hence the probability of a certain sequence of (r) outages will be  $p^r q^{n-r}$ . Using the rule of combinations and permutations, there are  $\frac{n!}{r!(n-r)!}$  (written as  $\binom{n}{r}$ ), equally probable sequences, in which (r) outages can occur in a system of (n) units. The probability of outage of the various combinations of (n) units is given by the binomial expansion:

$$(q+p)^n = q^n + \frac{n!}{(n-1)!} q^{n-1}p + \frac{n!}{2!(n-2)!} q^{n-2}p^2 + \dots + q^{n-r} p^r \dots + p^n$$

$$= q^n (1^n) q^{n-1}p + (2^n) q^{n-2}p^2 + \dots + (r^n) q^{n-r}p^r + \dots + p^n$$

where (r) is the number of units out of service at the same time due to forced outages.

**4. Poisson Distribution Method:**

The poisson distribution represents the probability that an isolated event occurs a certain number of times in a given interval of time when the rate of occurrence is fixed. The occurrence of the events must be affected by chance only such that the information about the position of one event is of no help in predicting the position of any other specific event. Furthermore data on one interval of time is of no help in predicting how many events will occur in any other interval. In other words, if the poisson distribution will mathematically represent a random phenomenon, the following conditions must be fulfilled:

1. The probability, that (K) random events occur during an interval (T) (between t and t+T) depends on (K) and (T) only and not on (t).
2. The probability that (K) random events occur between (t) and (t+T) does not depend on the number of random events observed before.
3. The probability of more than one random event in any interval of length (t) approaches zero faster than (t).

Derivation from binomial distribution:

The probability that an event succeeds or occurs (r) times in (n) trials is:

$$P_r = \frac{n!}{r!(n-r)!} p^r q^{n-r}$$

If (p) is very small and (n) is very large compared with (r) the above equation tends to a special form which is the poisson law of probability. Let the most probable number of occurrences (np) be some number

$$\lambda = np$$

$$\text{then } p = \frac{\lambda}{n}$$

$$p_r = \frac{n!}{r!(n-r)!} \left(\frac{\lambda}{n}\right)^r \left(1 - \frac{\lambda}{n}\right)^{n-r}$$

if  $n \rightarrow \infty$  and (r) is finite, then

$$p_r = \frac{n^r}{r!} \left(\frac{\lambda}{n}\right)^r \cdot e^{-\lambda} = \lambda^r \frac{e^{-\lambda}}{r!}$$

Substitute by (nP)

$$p_r = \frac{(nP)^r e^{-nP}}{r!}$$

which is the probability of (r) successes in (n) trials. The mean number of occurrences of an event per unit of time is  $\lambda = nP$  and the standard deviation of the number of events is:

$$\sigma = \sqrt{nP}$$

Thus the mean and the variance are equal

$$\lambda = \sigma^2 = nP$$

The poisson distribution can thus be used as an approximation of the binomial distribution when the sample size is large and the probability of success ( $P$ ) is small. Although the poisson distribution is derived from the binomial distribution it should be considered as a separate distribution. Thus the binomial and poisson distributions are two different distributions which give similar numerical results for large ( $n$ ) and small ( $P$ ). The peak of the poisson distribution is near ( $\lambda$ ) and the symmetry about the peak begins to develop at large values of ( $\lambda$ ).

### 5. Monte Carlo Method:

#### 5.1. Determination of the Forced Outage Probability of the Generating Capacities:

The application of Monte Carlo technique [6] to calculate the forced outage probability of the generating capacities in any power system can be detailed in the following:

- From statistical data on the peak load demand, a number of peak load days which characterise the pattern of the daily peak load over a year has to be chosen. In this paper the peak load of 48 days characterising the pattern of the daily peak load in the UPS of Egypt has been selected and used.
- Usually the yearly peak load is known and the other 47 peak loads are obtained knowing the statistical ratio of their peaks to that of the yearly peak.
- For the year to be studied the peak load level has to be determined.
- Each of the ratios is multiplied by the load level. This will determine the peak loads of the year considered. These peak loads are numbered.
- The generating capacities available to supply the loads during the year under consideration are arranged and a sequential number was given to each unit. The rate of forced outage  $P$  and the rated capacity ( $C$  rated) of each unit was specified and had the same number as that of the relevant unit.
- Random number generator is run to choose one of the peak loads  $P_L$  peak.
- The key features in a Monte Carlo simulation are the generation of a series of random variable with specified probability densities. The method used in this work to generate a series of random numbers is the congruential method. We consider the recursion relation.  

$$X_i = aX_{i-1} + c \pmod{m}$$
 where  $X_i =$   $i$ th pseudorandom number  
 $a, c =$  integers between 0 and  $m-1$   
 $m =$  quantity which sets repetition period.
- The operation  $X_i \pmod{m}$  is defined as the remainder which results when  $aX_{i-1} + c$  is divided by  $m$ . The maximum period obtainable is  $m$  digits, but shorter periods are also possible if  $m$ ,  $a$  and  $c$  are chosen without restriction.
- The first generating unit is considered, random number generator run, the resultant number is compared with the rate of forced outage of this unit  $P_i$ . If the random number is higher than  $P_i$  the unit is available, if not, the unit is considered not available.
- The same procedure is repeated with all other units.
- The summation of the available generating capacities  $\Sigma C$  rated was obtained and also  $\Sigma C$  out capacities are obtained.  $\Sigma C$  rated is then compared with  $P_L$  peak.
- If  $\Sigma C$  rated  $\geq P_L$  peak this implies running with sufficient generating capacity and  $\Sigma C$  rated -  $P_L$  peak = excess running capacity.
- If  $\Sigma C$  rated  $< P_L$  peak a generating power shortage results and  $P_L$  peak -  $\Sigma C$  rated = deficit power.
- The above procedure is repeated a large number of times (1000 trials).

#### 5.2. Determination of Reserve Requirements Using Continuous Random Variable Distribution:

The static reserve requirements can be determined using the results obtained by the previous method. The forced outage capacities obtained in the different  $N$  trials are arranged in rank and the grouped frequency table is easily calculated using a special designed computer program. The cumulative probability

is calculated as follows:

As the cumulative probability of zero forced outage = 1, it was assumed that the cumulative probability of the mid point of the first class interval is equal to one and the cumulative probability of the 2<sup>nd</sup> class interval  $P_2 = 1 - f_1/N$

where  $f_1$  is the class frequency of the first interval

$N$  is the total number of observations (trials).

The cumulative probability of the 3<sup>rd</sup> interval  $P_3 = P_2 - f_2/N$  and so on. If a curve is plotted to represent the cumulative probabilities for the forced outage capacities, the examination of the curve showed that these data points may be fitted to an exponential equation. The exponential equation is represented by:

$$P(x) = e^{mx}$$

where  $P(x)$  = cumulative probability for  $x$  MW

$x$  = forced outage capacity in MW

$m$  = constant to be determined

$$\ln P(x) = mx$$

Let in  $P(x) = Y$

$$Y = mx$$

The last equation represents the equation of a straight line which passes through the origin. This straight line should pass through the point  $(\bar{X}, \bar{Y})$  hence

$$m = \frac{Y - \bar{Y}}{X - \bar{X}}$$

where  $\bar{Y}$  is the mean of  $\sum_{i=1}^K Y_i$

$\bar{X}$  is the mean of  $\sum_{i=1}^K X_i$

$K$  is the number of class intervals.

To determine the value of  $m$ , apply the least square method by minimizing the sum of the squares of the residuals,  $m$  has the value.

$$m = \frac{\sum (Y_i - \bar{Y})(X_i - \bar{X})}{\sum (X_i - \bar{X})^2}$$

## 6. Calculation of Static Reserve Requirements:

### 6.1. Application to the Egyptian unified power system (UPS)

The above method was applied to compute the forced outage probability of the different generating capacities of the Egyptian unified power system during the year 1990. Static reserve capacity is evaluated. The data source is the reports issued by the National Dispatch Center. A computer program has been designed, tested and applied to carry out the study calculations. The input to the program includes the ratio of 48 daily peak loads characterizing the pattern of the daily peak load in the UPS, the load level for the year 1990 (6940 MW) and the data about the available generating capacities. This data includes the net available power of each unit taking into consideration the effect of age, the method of feeding the auxiliaries and the forced outage type and rate of each unit. Table 1 shows the forced outage rate of the different UPS generating units in the year 1990.

Zone	Station name	Unit installed capacity MW	Number of units	Forced outage rate
Cairo	Shobra El-Kheima	315	4	0.5
	Cairo West	75	4	2.1
	Cairo South (TH)	55	4	1.2
	Cairo South (GT)	110	2	2.0
	Cairo North	18	4	0.8
	Tebbin	15	3	3.1
	Tebbin	20	2	4.5
	Helwan	20	5	1.7
	Wadi Hoff	28	3	0.1
Delta	Kafr Dawar	90	3	9.0
	Damanhour	40	5	5.6
	Talkha	27	5	1.2
	Talkha	24	8	0.6
	Mahmoudia	33	12	0.3
	Damanhour	25	4	0.8
	Damietta	125	6	3.0
	Damietta	125	3	3.0
Alex	Abu Kir	225	5	7.4
	Suif	17	4	1.4
	Suif	32	7	1.2
Canal	Abu Sultan	150	4	1.5
	Ataka	225	4	3.4
	Suez	45	4	9.0
	Shahab	33	3	0.4
	Port Said	23	2	1.1
	Ismailia	20	1	0.7

Table (1): Forced outage rate of the different UPS generating units in the year 1990.

The ratio of 48 daily peak loads to the yearly peak load for the year 1990 is given in Table 2 and 3. The output of the program includes in addition to the input data, the predicted daily peak loads, the corresponding excess running capacity or deficit and the forced outage capacity in each one of the (N) trials. The actual quantity of static reserve is dependent upon many factors. One of the most important factors is the desired level of reliability or the desired risk index or the loss of load probability (LOLP).

.922190	.938040	.934294	.932565	.922190	.923631
.929107	.939914	.945245	.943228	.948415	.941210
.945245	.926801	.931988	.932421	.939193	.920461
.938040	.923919	.933429	.951009	.922190	.960231
.937608	.927233	.932277	.925216	.939709	.963401
.965418	.958790	.962536	.979251	.975937	.969452
.975793	.985793	.978674	.971326	.991354	.996686
.993228	.987752	.995101	.980692	.991066	1.00000

Table (2): Ratio of 48 daily load to yearly load for the year 1990

6400	6510	6484	6472	6400	6410
6448	6523	6560	6546	6582	6532
6560	6432	6468	6471	6518	6388
6510	6412	6478	6600	6400	6664
6507	6435	6470	6421	6522	6686
6700	6654	6680	6796	6773	6728
6772	6841	6792	6741	6880	6917
6893	6855	6906	6806	6878	6940

Table (3): M.W. values for the year 1990

The factors which affect the LOLP in the power systems, assuming that the transmission system is supposed to be infinitely safe and of sufficient capacity, are: the annual daily peak loads, the capacities, the number, the location and the arrangement of the steam turbines and boilers and the time required to start each of them, the maintenance schedules of the hydro capacities, the forced outage experience of the hydro units, the gas units, the steam turbogenerators and boilers and at last the economic considerations. A calculated value for the LOLP or the risk index in the Egyptian UPS is not available, so different assumed values based on the system behaviour and the past experience have been used to determine the reserve requirements. Two sets of results are obtained using either:

- The calculated forced outage rate of the different generating units of the Egyptian UPS, or
- The generic data for the forced outage rate of the generating units.

In this case the assumed values are:

- 4% forced outage rate for the steam units,
- 6% forced outage rate for the gas units,
- 1% forced outage rate for the hydro units.

A prototype program is implemented which combines the proceedings derived in sections 3.4 and 5. The program was implemented in Fortran on a PC.

### 6.2 Generic Data for the Forced Outage Rates:

The reserve required to secure 0.999 reliability (LOLP  $\approx$  0.001) for the UPS of Egypt in 1990 is at first calculated. Then different LOLP values have been assumed and accordingly reserve requirements are determined. The values used for the LOLP are:

- One day every three years
- One day every two years
- Two days every three years, and
- One day every year.

Fig. 1 shows the variation of the reserve values (in percentage of the peak load of 6940 MW for the year 1990) with the assumed range of LOLP values of one day and three days every three years. The maximum value calculated is 17.25% for a loss of load probability of one day every three years. Table 4 represents the grouped frequency table for the forced outage capacities data during the year 1990 for a LOLP of one day every two years (LOLP = 0.0014).

Class interval	Class midpoint M.W.	Class frequency	Cummulative Probability
0-100	50	70	1.00000
100-200	150	165	0.9285
200-300	250	197	0.7599
300-400	350	217	0.5587
400-500	450	117	0.3371
500-600	550	94	0.2175
600-700	650	54	0.1215
700-800	750	41	0.0664
800-900	850	14	0.0245
900-1000	950	8	0.0102
1000-1100	1050	1	0.002
1100-1200	1150	0	0.001
1200-1300	1250	1	0.001

Table (4) Grouped frequency table for the forced outage capacities data during the year 1990

Sensitivity analysis of the assumed forced outage rates of the different generating capacities has been carried out. Increasing or decreasing the forced outage rate of the different units by 10% of their initial value affects the value of the estimated reserve by 0.7% and 0.4% respectively. To study if there is an effect of the initial value of the random number chosen (IRN) on the results obtained, different values for IRN of 0.1319, 1.1319, 3.1319 and 5.1319 are used. The estimated value of reserve required to secure 0.8886 reliability does not differ in any case by more than 1.2% of the value calculated for the value of 0.1319 used in the paper.

### 6.3 Actual Forced Outage Rates:

Different values of LOLP are used to calculate the value of the reserve required to secure different reliability levels for the Egyptian UPS in 1990. The same values of LOLPs used with the generic data and the calculated reserve values are shown in Fig. 2. The maximum value of the reserve for the assumed range of LOLP values is 16.4% of the peak load for the year 1990. This value differs by about 0.8% from that in the case of generic data. The effect of the class interval used in the grouped frequency table is considered. Changing the class interval from 10 to 50, 100 and then to 120 shows that a class interval of 100 is suitable and no change occurs when the interval changes from 100 to 120. Fig. 3 is a sample of the cumulative probability curve obtained in one of the study cases. The effect of the number of trials on the calculated value of reserve is considered. Table 5 presents the variation of the reserve versus the number of trials in a class interval of 100, LOLP = 0.002 and IRN = 0.1319. The calculated values of reserve seem to be nearly independent of the number of trials if 1000 trials are used. In the whole paper 1000 trials are used.

Number of trials	Reserve MW
200	1274
400	1146
1000	1025
2000	1028
5000	1011
10000	1023

Table (5): Effect of number of trials on the calculated values of reserve



It can be concluded that the maximum calculated reserve value for the Egyptian UPS is 17.25% of the peak load (6940 MW in the year 1990) assuming a LOLP of one day every three years and assuming generic data for the forced outage rates of the different generating units. The actual forced outage rates for the UPS units differ considerably for some units from the used generic values. For example the calculated forced outage rates for Kafr Eldawar steam units are 12% higher. Although these differences occur, there was no remarkable effect on the final calculated reserve values.

#### 6.4 Calculation of Operating Reserve Requirements:

The operating reserve requirements are evaluated using both the binomial distribution and poisson methods. The cumulative probabilities of forced outage of the UPS generating units using these methods are given in Tables 6 and 7 respectively. If a LOLP of 0.00054 i.e. of one day every five years is used, the value of the operating reserve using the binomial distribution and poisson methods is 860 and 730 MW respectively.

Capacity <sup>(*)</sup> MW	Probabilities of forced outage capacity larger than capacity <sup>(*)</sup>
0.0	0.05459
60	0.03716
120	0.02534
180	0.02027
240	0.02176
300	0.01516
360	0.00775
420	0.00518
480	0.03697
540	0.00221
600	0.00120
660	0.00057
720	0.00032
780	0.00017
840	0.00008
900	0.00004
960	0.00002
1020	0.00001
1080	0.00000

Table (6) Cumulative probabilities of forced outage of the UPS generating units using the binomial distribution method

Capacity <sup>m</sup> MW	Probabilities of forced outage capacity larger than capacity <sup>m</sup>
0.00	0.04354328
67.61	0.13646462
135.23	0.21384007
202.84	0.22339159
270.46	0.17502731
338.07	0.10970712
405.69	0.05730368
473.30	0.02565568
540.92	0.01005061
608.53	0.00349985
676.15	0.00109685
743.76	0.00031250
811.37	0.00008162
878.99	0.00001968
946.60	0.00000440
1014.22	0.00000092
1081.83	0.00000018
1149.45	0.00000000

Table (7): Cumulative probabilities of forced outage of the UPS generating units using poisson method

### 7. Conclusions and Recommendations:

In this paper, the forced and planned outage events of the Egyptian unified power system generating units are collected, reviewed and analysed.

40% of the forced outage events of the steam units are due to boiler problems. The average outage hours due to boiler problems is in general higher than those of the other system problems except the turbine problems which are equal. The condenser problems caused 15.5% of the total forced outage events of the steam units which requires further study of the boiler and condenser problems in order to reduce the number of outage events and the average outage hours.

The unified power system problems caused 8.7% of the total outage events of the steam units and 11% of the total forced outage events of the gas turbine units.

Only 7 forced outage events for the steam units and 11 events for the gas turbine units are recorded due to human errors during the study period.

No data concerning the fire events in the unified power system is available from the power station or the reports therefore they were not included in the study.

Based on the way of recording the outage events of the generating units and from the application and comparison, the study recommends the following:

- Use of the attached form for recording the outage events of the generating units and its application for a period of one year.
- Further study of the boiler and condenser problems in order to reduce the number of outage events and the average outage hours.

### 8. References:

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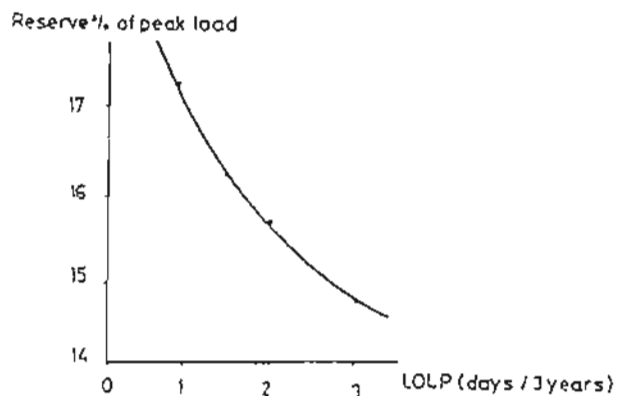


Fig. 1: Reserve values for the Egyptian UPS generating units using genetic data.

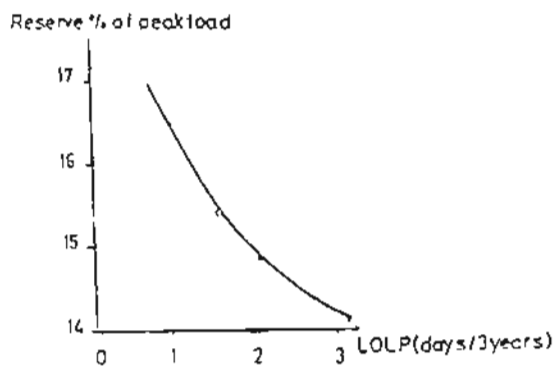


Fig. 2: Reserve values for the Egyptian UPS generating units using actual calculated values

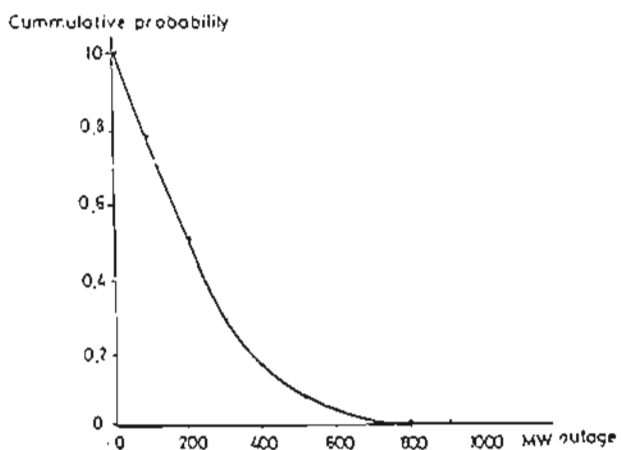


Fig. 3: Cumulative outage-probability curve