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COMBINED POISEUILLE- COUETTE STATIONARY FLOW FOR A FLUID OF GRADE TWO WITHIN THE ANNULAR REGION BETWEEN TWO ECCENTRIC TUBES

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ABSTRACT

The flow of a viscoelastic fluid of grade two which performs a combined Poiseuille -Couette flow in the annular region between two eccentric tubes is investigated . A concised review of the first -order solution (3,7) is given in order to form a suitable basis for the present calculation.

The results of the present work suggests a major modification of the eccentric-cylinder rheometer (1) which is based on the pure Couette flow. From the practical point of view , a rheometer based on the combined Poiseuille-Couette flow can be realized and is expected to give more information than the present one.

Key words : Viscoelastic fluid, Eccentric tubes ,Poiseuille - Couette flow , Secondary flow.

1. INTRODUCTION

In an extensive theoretical study of the steady state motion of a fluid of grade two in the annular region between two eccentric tubes, the velocity field due to superposition of rectilinear shearing, Poiseuille and Couette flow is investigated (3,7).The calculations are carried out within the limit of the retarded motion expansion (4). Up to the first-order (viscous or Navier-Stokes fluid), the three motions create three independent contributions; namely $W_{1s}(\zeta,\eta)$ for simple shearing, $W_{1p}(\zeta,\eta)$ for Poiseuille flow and $\underline{u}_1(\xi,\eta)$ for Couette flow. Provided each of the afore-mentioned three motions is considered separately, the velocity field for a fluid of grade two coincides with that of the Navier-Stokes fluid. However, if any of the two axial motions is superimposed on the Couette flow, a second-order contribtion $W_2(\zeta,\eta)$ takes place. This second-order term results as an interaction between axial and rotational flow. The second-order term $W_2(\zeta,\eta)$ is already calculated (3,7) for the combination of simple shearing and Couette flow; i.e. for the so called helical flow.

In the work (3,7), the authers suggested the application of this boundary value problem for the construction of a rheometer which allows the determination of some of the second-order (elastic) constants besides the first-order (Newtonian) viscosity.

This suggestion is realized partially in a very important and fundamental contribution to Rheometry (1,2). In this work a rheometer, termed a new eccentric cylinder rheometer, is constructed on the basis of the pure Couette flow. The authers corrected the stream function for the first order Couette flow given in (3,7). They showed that the rheometer is capable of determining the shear viscosity μ and the second- order elastic constant α_1 in a convenient way and only by one and the same set up. Due to the impossibility of realizing the combination of simple shearing and Couette flow, a further development of the eccentric cylinder rheometer is not possible. However, the combined Poiseuille-Couette flow is much easier to be realized. Hence, the aim of the present work is to determine the second order velocity field $W_2(\zeta,\eta)$ hen Poiseuille flow is superimposed on Couette flow.

2. Formulation Of The Problem

The fluid of grade two is assumed to perform isochoric and steady motion in the annular region between two eccentric cylinders of radii R_1, R_2 $(R_1 < R_2)$ and of infinite lengths. The two axis of the cylinders stand parallel to each other and to the X_3 - axis. The geometry of the problem suggests the use of cylindrical bipolar coordinates (ζ,η,z). This system of coordinates is generated from the rectangular system of coordinates (X_1, X_2, X_3) by the conformal transformation (5).

$$W = 2\,th^{-1} z/c \qquad Z = X_3 \qquad (1)$$

where

$W = \zeta + i\eta$ and $Z = x_1 + ix_2$. Eq (1) is equivalent to the real transformation

$$x_1 = h\, sh\zeta \quad ; \quad \zeta = \tanh^{-1}\left[\frac{2cx_1}{c^2 + x_1^2 + x_2^2}\right]$$

$$x_1 = h \sin\eta \quad ; \quad \eta = \tan^{-1}\left[\frac{2cx_2}{c^2 - x_1^2 - x_2^2}\right]$$

$$x_3 = z \quad ; \quad z = x_3 \quad , \qquad (2)$$

where $h = c/(ch\,\zeta + \cos\eta)$ is the scale factor for the bipolar coordinates. The cross-section of the flow region is shown in Fig (1) where ζ_1 and ζ_2 represent the cross-sections of the inner and outer cylinders; respectively .

The first-order velocity field calculated in (3,7)is given by the expression ,

$$\underline{V} = \hat{Z}[W_{1s} + W_{1p}] + \underline{u}_1(\zeta, \eta), \tag{3}$$

where the three contributions W_{1s}, W_{1p} and $\underline{u}_1$ are explained as follows

(i) The simple shearing created by the axial sliding of the inner tube with constant velocity $W_o \hat{z}$ induces a velocity field given by

$$W_{1s}\hat{Z} = W_o \frac{\zeta-\zeta_2}{\zeta_1-\zeta_2}\hat{z} + 0(W_o^2) \tag{4}$$

(ii) The Poiseuille flow due to the constant pressure gradient "a" in the axial direction creates the velocity field

$$W_{1p}\hat{z} = \frac{ac^2}{2\mu}\hat{z}\left\{\frac{\zeta-\zeta_2}{\zeta_1-\zeta_2}\coth\zeta_1 + \frac{\zeta_1-\zeta}{\zeta_1-\zeta_2}\coth\zeta_2 - \frac{ch\zeta}{ch\zeta+\cos\zeta} + \right.$$

$$+\sum_{n=1}^{\infty}\frac{2(-1)^n\cos n\eta}{sh\, n(\zeta_1-\zeta_2)}\left[\coth\zeta_1 e^{-n\zeta_1} sh\, n(\zeta-\zeta_2)\right.$$

$$\left.\left. + \coth\zeta_2 e^{-n\zeta_2} sh\, n(\zeta_1-\zeta)\right]\right\} + o(\alpha^2)$$

(5*)

(iii) The Couette flow due to the rotation of the inner tube about its own axis with constant angular velocityir Ω is given by

$$\underline{u}_1 = -\hat{z}\wedge\nabla\psi_1 = h^{-1}[\hat{\zeta}\psi_{1,\eta} - \hat{\eta}\psi_{1,\zeta}], \tag{6a}$$

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* The final solution W_{1p} given in (3.7) includes a mistake which is corrected here

Where

$$\Psi_1 = Qh(\zeta,\eta)\{\wedge[(\zeta_1-\zeta)\text{sh}\delta\ \text{sh}(\zeta_1-\zeta)$$
$$-\delta(\zeta-\zeta_2)\text{sh}(\zeta_1-\zeta)] + \frac{\text{ch}\,\zeta_1}{\delta\,\text{sh}\,\delta}(\zeta-\zeta_2)\ \text{sh}(\zeta-\zeta_2)$$
$$+\ \frac{\cos\eta}{\delta\,\text{ch}\,\delta - \text{sh}\,\delta}[(\zeta-\zeta_2)\ \text{ch}\delta - \text{ch}(\zeta_1-\zeta)\text{sh}(\zeta-\zeta_2)]\}$$
$$+O(a^2) \qquad (6b)$$

where

$$\wedge = [\text{sh}^2\delta - \delta^2]^{-1}[\frac{\text{sh}\,\delta\,\text{ch}\,\zeta_1 + \delta\,\text{ch}\,\zeta_2}{\delta\,\text{sh}\,\delta} + \frac{c\Omega}{Q\text{sh}\,\zeta_1}]$$

The rate of flow , c Q , about the inner tube calculated per unit length is related to the angular velocity Ω by the relation

$$Q = \frac{c\,\Omega}{\text{sh}\zeta_1}[\frac{(\delta\,\text{ch}\delta - \text{sh}\delta)\ (\delta\,\text{sh}\zeta_1 - \text{sh}\,\delta\ \text{sh}\,\zeta_2)}{\text{sh}\,\delta\,[\text{ch}\sigma\,(\delta\,\text{ch}\,\delta - \text{sh}\,\delta) + \text{sh}\,\delta\,\text{ch}\,\delta - \delta}] \qquad (6\text{-c})$$

where $\delta = \zeta_1 - \zeta_2$ and $\sigma = \zeta_1 + \zeta_2$.

The equation determining the second - order solution is the Poisson equation given by

$$\nabla^2 w_2 + \frac{\alpha_1 + \alpha_2}{\mu}\nabla.\ [\nabla w_{1p}\ (\nabla \underline{u}_1 + (\nabla \underline{u}_1)^T)] = 0,$$

$$W_2 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \text{ at } \zeta = \begin{Bmatrix} \zeta_1 \\ \zeta_2 \end{Bmatrix}, \qquad (7)$$

where $\underline{u}_1$ is the rotational flow defined by Eqs . (6a - 6c) and W_1 is ,in the present case , the Poiseuille flow defined by Eq (5) .

The constants μ, α_1 and α_2 are the viscosity and the second - order material coefficients according to the constitutive equation

$$\underline{\underline{T}} = -P\underline{\underline{I}} + \mu\underline{\underline{A}}_1 + \alpha_1\underline{\underline{A}}_2 + \alpha_2\underline{\underline{A}}_1^2; [3], \tag{8a}$$

where

$$\underline{\underline{A}}_1 = \nabla\underline{u}_1 + (\nabla\underline{u}_1)^T. \tag{8b}$$

and

$$\underline{\underline{A}}_2 = \frac{D\underline{\underline{A}}_1}{Dt} + \nabla\underline{u}_1.\underline{\underline{A}}_1 + \underline{\underline{A}}_1.(\nabla\underline{u}_1)^T \tag{8c}$$

In Eq .(8a) $\underline{\underline{T}}$ is the stress tensor ,P is the hydrostatic pressure and $\underline{\underline{I}}$ is the unit tensor . It is obvious that $\underline{\underline{A}}_2$ and $\underline{\underline{A}}_1^2$ are second -order terms in the velocity field.

Eq (7) implies that W_2 is , as mentioned before different from zero only; if both W_1 and $\underline{u}_1$ are different from zero.

3. Solution of the problem

The solution given in (3,7) for the case of helical flow was performed in terms of the proper Green function, the simplicity of the density function left the integration over the Green function within tractable limits. However, the density function $\nabla.\left\{\nabla W_1.\left[\nabla\underline{u}_1 + (\nabla\underline{u}_1)^T\right]\right\}$ with W_1 given by Eq .(5) is so complicated that the integration process is associated with serious difficulties. Fortunately, a close investigation of the density function shows that a solution for Eq . (7) can be obtained directly .

Consider,

$$\begin{aligned}
&\nabla.\{\nabla W_1.[\nabla \underline{u}_1 + (\nabla \underline{u}_1)^T]\} \\
&= \nabla.[\nabla W_1.\nabla \underline{u}_1 + \nabla \underline{u}_1.\nabla W_1] \\
&= \nabla.[\nabla(\underline{u}_1.\nabla W_1) - \underline{u}_1.\nabla\nabla W_1 + \nabla W_1.\nabla \underline{u}_1] \\
&= \nabla^2[\underline{u}_1.\nabla W_1) - \nabla \underline{u}_1 : \nabla\nabla W_1 - \underline{u}_1.\nabla(\nabla^2 W_1) \\
&+ \nabla\nabla W_1 : \nabla \underline{u}_1 + \nabla W_1.\nabla(\nabla.\underline{u}_1) \\
&= \nabla^2(\underline{u}_1.\nabla W_1). \qquad (9)
\end{aligned}$$

Since $\nabla.\, \underline{u}_1 = 0$ by the equation of continuity and $\underline{u}_1.\nabla(\nabla^2 W_1) = \underline{u}_1.\nabla(-\frac{\alpha}{\mu}) = 0$. Substituting from Eq (9) into Eq. (7) we obtain the following equation

$$\nabla^2\left[W_2 + \frac{\alpha_1 + \alpha_2}{\mu}\underline{u}_1.\nabla w_1\right] = 0\,, \qquad (10)$$

which has the solution .

$$W_2 = -\frac{\alpha_1 + \alpha_2}{\mu}\underline{u}_1.\nabla w_1 + W_2^{(h)},$$

where $W_2^{(h)}$ is the solution of the Laplace equation $\nabla^2 W_2 = 0$.

Since $\underline{u}_1.\nabla W_1$ satisfies the boundary conditions, then $W_2^{(h)}$ is equal to zero. Hence ,

$$\begin{aligned}
W_2 &= -\frac{\alpha_1 + \alpha_2}{\mu}\underline{u}_1.\nabla W_1 \\
&= -\frac{\alpha_1 + \alpha_2}{\mu}h^{-1}(u_{1\zeta} W_{1,\zeta} + u_{1\eta} W_{1,\eta}). \qquad (11)
\end{aligned}$$

$u_{1\zeta}$ and $u_{1\eta}$ are to be calculated from ψ_1,Eq (6b), and $W_{1,\zeta}$ and $W_{1,\eta}$ are to be calculated from W_{1p} ; Eq. (5).

Therefore,

$$u_{1\zeta} = \frac{Qh}{C} \sin \eta \{\wedge[(\zeta_1 - \zeta) \,\mathrm{sh}\, \delta \,\mathrm{sh}(\zeta-\zeta_2) - \delta(\zeta-\zeta_2)\mathrm{sh}(\zeta_1-\zeta)]$$

$$-\frac{\mathrm{ch}\,\zeta_1}{\delta \,\mathrm{sh}\, \delta}(\zeta-\zeta_2)\,\mathrm{sh}\,(\zeta-\zeta_2) - \frac{\mathrm{ch}\,\zeta}{\delta\,\mathrm{ch}\,\delta - \mathrm{sh}\,\delta}[(\zeta-\zeta_2)\,\mathrm{ch}\,\delta$$

$$-\mathrm{ch}\,(\zeta_1-\zeta)\,\mathrm{sh}\,(\zeta-\zeta_2)]\} \qquad (12a)$$

$$u_{1\eta} = \frac{Qh}{C}\{\frac{h^{-1}}{C}\{\wedge[\mathrm{sh}\,\delta \,\mathrm{sh}(\zeta-\zeta_2) + \delta\,\mathrm{sh}\,(\zeta_1-\zeta)]$$

$$-\frac{\mathrm{ch}\zeta_1}{\delta \,\mathrm{sh}\,\delta}\mathrm{sh}(\zeta-\zeta_2) + \frac{\mathrm{ch}(\sigma-2\zeta)-\mathrm{ch}\delta}{\delta\,\mathrm{ch}\,\delta - \mathrm{sh}\delta}\cos\eta\}$$

$$-(\mathrm{ch}\zeta_2 + \mathrm{ch}(\zeta-\zeta_2)\cos\eta)\{(\zeta_1-\zeta)^\wedge \mathrm{sh}\delta$$

$$+\frac{\zeta-\zeta_2}{\delta\,\mathrm{sh}\delta}\mathrm{ch}\zeta_1\} - (\mathrm{ch}\zeta_1 + \mathrm{ch}(\zeta_1-\zeta)\cos\eta)\delta^\wedge(\zeta-\zeta_2)$$

$$+\frac{\mathrm{sh}\zeta\cos\eta}{\delta\,\mathrm{ch}\,\delta - \mathrm{sh}\delta}[(\zeta-\zeta_2)\mathrm{ch}\,\delta - \mathrm{ch}(\zeta_1-\zeta)\mathrm{sh}(\zeta-\zeta_2)]\}. \qquad (12b)$$

$$W_{1,\zeta} = \frac{-a\,c^2h^{-1}}{2\mu}\left[-\frac{\mathrm{cth}\zeta_2 - \mathrm{cth}\zeta_1}{\delta} + \frac{\mathrm{sh}\zeta\cos\eta}{(\mathrm{ch}\zeta + \cos\eta)^2} + \sum_{n=1}^{\infty}\frac{2n(-1)^n\sin\eta}{\mathrm{shn}\delta}\right.$$

$$\left.[\mathrm{cth}\zeta_1 e^{-n\zeta_1}\mathrm{chn}(\zeta-\zeta_2) - \mathrm{cth}\zeta_2 e^{-n\zeta_2}\mathrm{chn}(\zeta_1-\zeta)]\right]. \qquad (12c)$$

$$W_{1,\eta} = \frac{-a\,c^2h^{-1}}{2\mu}\left[\frac{\mathrm{ch}\zeta\sin}{(\mathrm{ch}\zeta + \cos\eta)^2} + \sum_{n=1}^{\alpha}\frac{2n(-1)^n\sin\eta}{\mathrm{shn}\delta}\right.$$

$$\left.[\mathrm{cth}\zeta_1 e^{-n\zeta_1}\mathrm{shn}(\zeta-\zeta_2) - \mathrm{cth}\zeta_2 e^{-n\zeta_2}\mathrm{shn}(\zeta_1-\zeta)]\right]. \qquad (12d)$$

Substituting Eqs (12a) - (12d) into Eq (11) we get the proper expression for $W_2(\zeta, \eta)$.

This expression is quite complicated. The construction of rheometers is frequently done for narrow annular width. Thus the previous expressions will be approximated for the case $\delta = \zeta_1 - \zeta_2 << \zeta_2$. Moreover, the change of variables

$$y = \frac{\zeta - \zeta_2}{\delta},$$

shows to be useful , because it reduces the range

$\zeta_2 \leq \zeta \leq \zeta_1$ to $0 \leq y \leq 1$.

Substituting the last transformation into Eq (12a) we get

$$u_{1\zeta} = \frac{Qh \sin \eta}{C}\{\delta^{\wedge}[(1-y)\text{sh}\,\delta\;\text{sh}\,\delta y - \delta y\,\text{sh}\delta\,(1-y)]$$

$$+\frac{\text{ch}\zeta_1}{\text{sh}\delta}\text{sh}\,\delta\,y - \frac{\text{ch}\zeta}{\delta\,\text{ch}\,\delta - \text{sh}\,\delta}[\delta y\,\text{ch}\,\delta - \text{ch}\delta\,(1-y)\text{sh}\,\delta y]\}.$$

Expanding the hyperbolic functions in terms of power series, and neglecting terms of $0(\delta^2)$ relative to one , then

$$u_{1\zeta} = \frac{Q\,\delta h(y,\eta)y^2(1-y)}{c}\sin\eta\left[\frac{c\Omega}{Q\text{sh}\zeta_1} + 2(1-y)\text{sh}\zeta_2\right]$$

With $u_{1\zeta} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ at $y = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$, (14a)

and Eq. (12b) reduces to

$$u_{1\eta} = \frac{yQh(y,\eta)}{c}\{(\text{ch}\zeta + \cos\eta)[\frac{\delta^4\wedge}{6}(3y-1) - \frac{\text{ch}\zeta_1}{\delta}$$

$$-\frac{6(y-1)}{\delta}\cos\eta] - (\text{ch}\zeta_2 + \text{ch}\delta\;y\cos\eta)\frac{\text{ch}\zeta_1}{\text{sh}\,\delta} + y\,(3-2y)\text{sh}\zeta\cos\eta\},$$

with $u_{1\eta} = \begin{Bmatrix} \Omega / \text{sh}\zeta_1 \\ 0 \end{Bmatrix}$ at $y = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$, (14b)

In the same manner where coth ζ is expanded according to the relation,

$$\coth\zeta=\coth(\zeta_2+\delta y)=\coth\zeta_2-\frac{\delta y}{sh^2\zeta_2},$$

which on substitution into Eq. (5) leads to

$$W_1=\frac{a\delta c^2}{2\mu\, sh^2\zeta_2}\sum_{n=1}^{\alpha}\frac{2(-1)^n e^{-n\zeta_2}}{shn\delta}[y\, e^{-n\delta y}shn\delta-e^{-n\delta}shn\delta y]\cos n\eta$$

with $w_1=\begin{Bmatrix}0\\0\end{Bmatrix}$ at $y=\begin{Bmatrix}1\\0\end{Bmatrix}$. (15a)

Therefore

$$W_{1,\zeta}=\frac{ac^2}{\mu\, sh^2\zeta_2}\sum_{n=1}^{\infty}\frac{(-1)^n\, e^{-n\zeta_2}}{sh\, n\delta}[(1-n\delta y)e^{-n\delta y}\, sh\, n\delta$$

$$-n\delta\, e^{-n\delta}chn\delta y]\cos n\eta.$$

with $W_{1,\zeta}\neq 0$ at $y=\begin{Bmatrix}1\\0\end{Bmatrix}$, (15b)

and

$$W_{1,\eta}=\frac{\alpha\,\delta^2 c}{\mu\, sh^2\zeta_2}\sum_{n=1}^{\alpha}n(-1)^{n+1}[y\, e^{-n\delta y}\, sh\, n\delta$$

$$-e^{-n\delta}\, sh\, n\delta y]\sin n\eta$$

(15c)

with $W_{1,\eta}=\begin{Bmatrix}0\\0\end{Bmatrix}$ at $y=\begin{Bmatrix}0\\0\end{Bmatrix}$.

Substituting by Eqs.(14a),(14b),(15b) and (15c) into Eq. (11), the approximated form of W_2 is given by

$$W_2 = -\frac{\alpha(\alpha_1+\alpha_2)Qcy}{\mu\, sh^2\zeta_2}\sum_{n=1}^{\alpha}(-1)^n e^{-n\zeta_2}\left\{\left[y(1-y)\begin{pmatrix} ch\zeta_1 + ch\zeta_2 \\ +\dfrac{\delta c\Omega}{Q sh\zeta_1}\end{pmatrix}\right.\right.$$

$$ch\zeta_1 - (3-2y)ch\zeta\left]\left[e^{-n\delta y}(1-n\delta y) - n\delta e^{-n\delta}\coth n\delta y\right]\right.$$

$$y\sin\eta\cos n\eta - \left((ch\zeta + \cos\eta)\left[\frac{3y-1}{2}\left(\frac{ch\zeta_1}{\delta} + \frac{ch\zeta_2}{sh\delta}\right.\right.\right.$$

$$\left.+\frac{c\Omega}{Qsh\zeta_1}\right) - \frac{ch\zeta_1}{\delta} - \frac{6(y-1)}{\delta}\cos\eta\Bigg] - (ch\zeta_2 + ch\delta y\cos\eta)\frac{ch\zeta_1}{sh\delta}$$

$$\left.\left.+y(3-2y)sh\zeta\cos\eta\right)\left(y e^{-n\delta y} - e^{-n\delta}\frac{sh\, n\delta y}{sh\, n\delta}\right)n\delta\sin n\eta\right\}$$

$$\text{with } W_2 = \begin{Bmatrix}0\\0\end{Bmatrix} \quad \text{at} \quad y = \begin{Bmatrix}1\\0\end{Bmatrix}. \tag{16}$$

DISCUSSION

Figure (2) shows the dependence of the normalized second - order velocity field,

$$W_2^* = \frac{-\mu W_2}{\alpha(\alpha_1+\alpha_2)Qc}.10^4$$

as function of the coordinate y , whereas the coordinate η surves as a parameter . The constants in Eq (16) are set $\zeta_1 = 1.1$, $\zeta_2 = 1$; i.e. $\delta = 0.1$. Figure (2) reveals that W_2^* possesses complicated behavior which will be discussed in the following points :

(i) Eq. (16) shows that

$$W_2^*(y,0) = W_2^*(y,\pi) = 0 \text{ such that } W_2^* = 0$$

on the line of centers of the two cylinders.

(ii) The analysis of the curves start at y =0 ; i . e., at the wall of the rotating inner cylinder. The curve representing $W_2^*(y,\pi/4)$ describes the second - order velocity on a line which crosses the annular width in the narrow region. On this curve W_2^* increases in magnitude with negative values up to a minimum at $y \approx 0.15$ then goes to zero at $y \approx 0.43$. Thus , in the interval $0 \le y \le 0.43$, $W_2^*(y,\pi/4)$ opposes the primary flow and slows down the velocity of the fluid. On the other hand , in the region $0.43 \le y \le 1$, $W_2^*(y,\pi/4)$ is positive with a maximum at $y \approx 0.65$. In this interval the resultant axial velocity is greater than the primary flow alone .

(iii) The curve $W_2^*(y,\pi/2)$ which still lies in the narrower region of the annular width shows the same behavior at the previous curve. However, $| W_2^*(y,\pi/2) | < | W_2^*(y,\pi/4) |$ on the whole interval $0 \le y \le 1$ except in the small intertval $0.375 < y < 0.45$ as $W_2^* \to 0$ and in the interval $0.8 \le y \le 1$.

(iv) The curve $W_2^*(y,3\pi/4)$, which lies in the wider region of the annular width, shows different behavior. Within the interval $0 \le y \le 0.24$ $W_2^*(y,3\pi/4)$ is positive with maximum at $y \approx 0.1$, such that the resultant axial velocity is greater than the primary velocity W_1^* . In the middle region $0.24 \le y \le 0.85$, $W_2^*(y,3\pi/4)$ is negative with minimum at $y \approx 0.53$. The motion of the fluid in this region is slower than the primary motion. Finally , in the interval $0.85 \le y \le 1$, $W_2^*(y,3\pi/4)$ is positive with maximum at $y \approx 0.92$. This behavior shows that the velocity distribution tends to be flatter than the distribution of the primary axial velocity.

(v) Close investigation of the construction of the eccentric cylinder rheometer (1,6) shows that its modification to include the Poiseuille axial flow is quite possible . However , to get the practical formulas to be applied in this case a careful calculation of the surface tractions, forces and torques at the wall of the outer cylinder is necessary .These calculations will be the subject of a further work.

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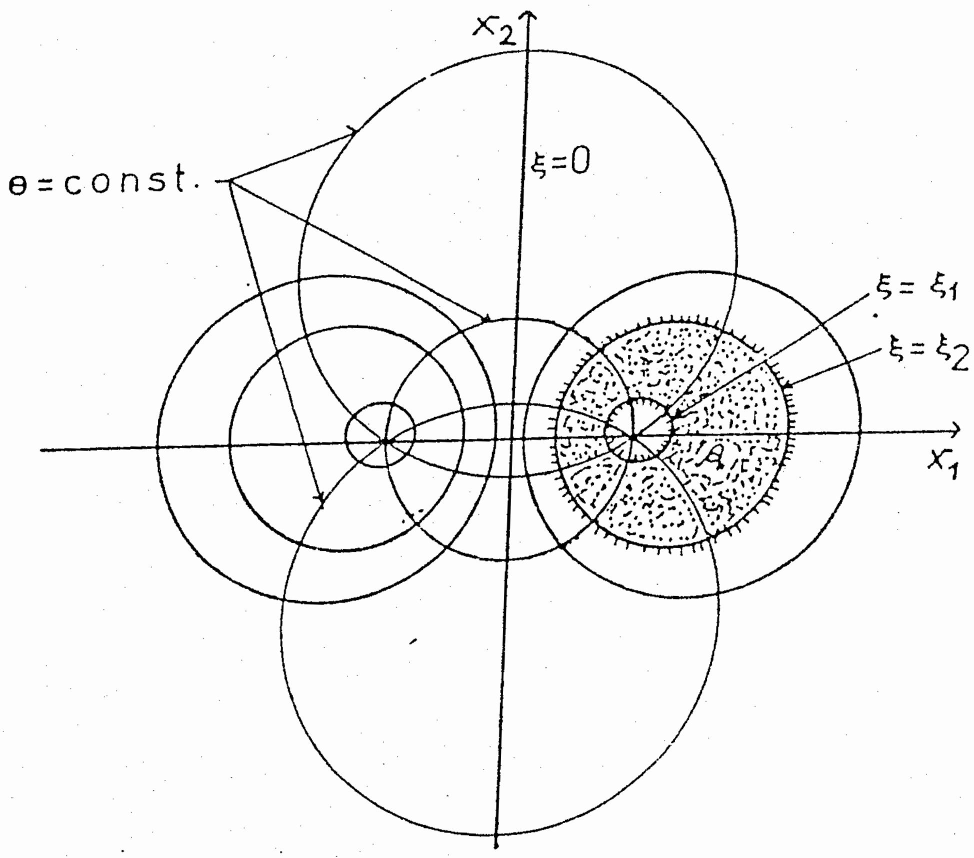

Fig. (1) . The cross-section of the annular region between two eccentric pipes in the x_1 x_2-plane, including a map of the bipolar coordinates in the plane.

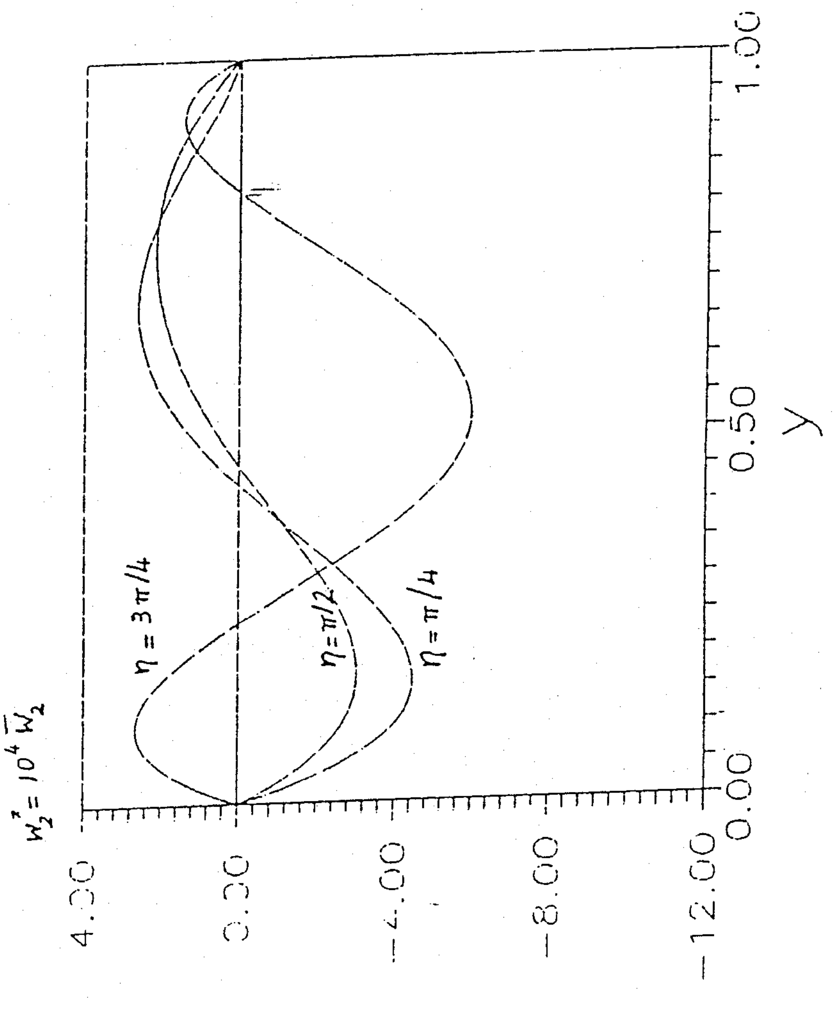

Fig.(2).Distribution of the normalized second-order velocity $W_2^*(y,\eta)$ as a function of y with η as a parameter.

سريان كويت وبوازيه المركب لسائل من الدرجة الثانية داخل الحيز المحصور بين اسطوانتين غير متحدتى المحور

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فى هذا البحث تم تعيين مجال السرعة من الدرجة الثانية-W_2- الناشئ من تدفق سوائل الدرجة الثانية تحت تأثير حركتين (سريان بوازى + سريان كويت) فى الفراغ الحلقى بين إسطوانتين لامركزيتين، ومن الجدير ذكره أن مسألة الشروط الحدية لاسطوانتين لامركزيتين قد تم دراستها فى بحث سابق وقد بنى على أساسها جهاز ريومترى يعتمد على الحركة الدورانية فقط. هذا وقد أجرى هذا البحث بغرض تطوير هذا الجهاز السابق وتوسيع مجال تطبيقه.