| Mansoura University<br>Faculty of Engineering<br>Eng. Math. and Phys.<br>Dept. | Final First Term<br>Exam.<br>2008-2009 | Mathematics (1).<br>Preparatory Year.<br>Time allowed: 3 Hours. |
| --- | --- | --- |

من فضلك إبدأ حل الجبر من إحدى جهتى ورقة الإجابة و التفاضل من الجهة الأخرى

## Algebra

### Question.1 [ 36 marks]

(a) Expand $(3 + \sqrt{x})^{-2}$ by the binomial series, state the convergence condition on $x$.

(b) Use mathematical induction to prove that for any positive integer $n$,

$$4^1 + 4^2 + 4^3 + \cdots + 4^n = \frac{4(4^n - 1)}{3}.$$

(c) Decompose the fraction $\dfrac{x-1}{x^2(x^2 + x + 1)}$ into its partial fractions.

(d) Find the equation whose roots are those of equation $x^4 + 8x^3 + 24x^2 + 32x = 0$ and each increased by 2, and then solve the two equations.

### Question.2 [ 36 marks]

(a) Suppose $A$ is $3 \times 3$ orthogonal matrix. If $B = 2A^{-1}$, find a value of $\det B$ ?

(b) Find $A^{-1}$ for $A = \begin{bmatrix} -1 & 3 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ using row operations method. Hence, or by any oth

method, solve the system $AX + 2B = C$, where $B = \begin{bmatrix} -2 \\ 3 \\ 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

(c) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & -7 \end{bmatrix}$.

# Differential Calculus

**Question 3.** *(36 marks)*

a) For $\ln y = \sin^{-1} x$ prove that

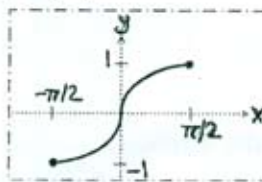$$(1 - x^2)y^{(n+2)} = (2n+1)x\, y^{(n+1)} + (n^2 + 1)y^{(n)}$$

b) Find

$$\lim_{x \to 0}(\cosh x)^{1/(e^x - 1)}$$

c) For the function $f(x) = \sin x$, $-\pi/2 \le x \le \pi/2$,

given in the figure

   i)  Sketch $f^{-1}(x)$, $\dfrac{1}{f(x)}$, $and\ f'(x)$

   ii)  Verify Mean value theorem for $f(x)$

d) Prove that the solution of the equation    $\sinh x = \sin \csc^{-1} e^x$

   is $x = \ln \sqrt{3}$


**Question 4.** *(36 marks)*

a) Find $y'$ if $\quad x^y = \dfrac{3^x \sin^3 x}{e^{\tan^{-1}\sqrt{x}}} + \text{sech} \cosh^{-1} x$

b) What value should $m$ assigned to make the function $f(x)$ continuous at $x = 0$,

where $f(x) = \begin{cases} x\ \csc x, & x < 0 \\ \ln(x + m), & x \ge 0 \end{cases}$

c) Find the Taylor series pansion of $f(x) = e^{\cos x}$ about $x = \pi/2$.

d) Prove that $\quad \tanh^{-1} x = \ln\left(\sqrt{\dfrac{1+x}{1-x}}\right)$, $\ |x| < 1$.

لجنة الممتحنين: ا.د. صلاح البحيري ا.د. مصطفي صالح    د. رضا عبده    د. ايهاب مصطفي