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FRACTURE STUDIES AND A PROCEDURE FOR K_{IIIC} DETERMINATION TO CONTROL AND PREVENT FAILURE OF ROTATING SHAFTS

دراسة تحليلية وتخطيط خطوات تقدير معامل الإطاقة لحماية الأعمدة الدوارة من الإنهيار

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ملخص البحث

نظرا لتعدد استخدام الاعمده الدواره في التطبيقات الصناعيه في المجالات الهندسيه المتعدده واهمية تشغيل هذه الاعمده في حدود الامان المسموح بها مع تجنب الانهيار الفجائي اثناء التشغيل تحت تأثير شدة كثافة تركيز الاجهادات التي قد تتخطى معامل الاطاقه للتشغيل الأمن.

فقد اهتم هذا البحث بدراسة سلوك المعدن المصنع منه الاعمده الدواره اثناء التشغيل لتجنب احتمالات الانهيار اثناء التشغيل لذلك فقد تم تخطيط هذه الدراسه على ثلاث محاور تشتمل على الاتي:

١-استنتاج معادلات تحليليه يمكن استخدامها لتقدير معامل شدة كثافة تركيز الاجهادات تحت تاثير الاحمال الخارجيه اثناء التشغيل .

٢-دراسة المعاملات التي تؤثر ويتحدد على ضوئها نظم التحكم في تنفيذ معامل تقدير شدة كثافة تركيز الاجهادات وسلوك الاعمده حتى الانهيار .

٣-اقتراح خطوات تقدير معاملات شدة كثافة تركيز الاجهادات الفعليه اثناء التحميل وكذلك تقدير معامل الاطاقه الذي يحدد المدى المسموح به لمعامل تقدير شدة كثافة تركيز الاجهادات لتجنب حدوث الانهيار الفجائي . وقد تم استنتاج معادلات رياضيه اقترح استخدامها لتقدير قيمة معامل شدة كثافة تركيز الاجهادات و المعاملات المؤثره في تحديد هذه القيمه . وباستخدام هذه المعادلات واتباع الخطوات المقترحه في البند الثالث امكن استنتاج منحنى المقاومه (R-curve) المتعارف عليه. وقد تم استخدم هذا المنحنى في تحديد معامل حد الاطاقه حتى يمكن تحديد الحد الاقصى المسموح به لمعامل تقدير شدة كثافة تركيز الاجهادات للاعمده الدواره لتحديد اجهادات التشغيل الأمن.

ولتأكيد فعالية المعادلات المقترحه تم معمليا قياس المعاملات المؤثره في تحديد سلوك المعدن ومقارنة تلك النتائج بالنتائج البيانيه والتي اظهرت تقاربا مقبولا.

ABSTRACT

When the particular combination of stress and crack size in a component reaches the critical stress intensity factor (fracture toughness), fracture will occur. Thus, the torsional mode fracture toughness K_{IIIC} is an important factor in the design and analysis of fracture of structure subjected to torsional mode loading (Mode-III).

It is the specific intent of investigated fracture control plan to establish the possible range of the torsional mode stress intensity factor denoted by K_{III}, that might be present throughout the life time of the component to ensure that the critical stress intensity factor denoted by K_{IIIC} (fracture toughness) for the material used is sufficiently large so that the component will have a safe life.

Procedure of elastic-plastic analysis has been developed for the determination of the fracture toughness in mode-III K_{IIIC} to control and prevent torsional mode fracture of rotating shafts.

The procedure investigated concerns: analytical and experimental stress analysis of pre-cracked bars, development of a control plan for evaluation of the critical stress intensity factor and resistance of materials to crack growth and fracture. The experimental investigation has been carried out to verify the analytical results, thereupon reasonable agreement is found, resulted in a better understanding of the possible fracture characteristics of the structure under consideration.

KEYWORDS
Fracture mechanics , stress intensity factor, fracture toughness, fracture control plan, torsional mode.

INTRODUCTION

In the case of brittle fracture, many of the fracture control guidelines have been followed to minimize the possibility of brittle fracture in structures. These guidelines include the use of member material with

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good fracture toughness. elimination or minimization of stress raisers. control of crack growth. proper inspection. etc When these general guidelines are integrated into specific requirements for a particular structure or component they will be the basic parameters for establishment of a fracture control plan.

A fracture control plan is therefore a specific set of recommendations developed for a particular component or structure and should not be indiscriminately applied to other structures.

Thereupon. the main objective for conducting this research work is to establish a fracture-control plan to prevent failure of rotating shafts. which may thus be used in a wide sector of industrial field. The fracture mechanics principles can be used in developing a new procedure for determination of the basic element of a fracture-control plan The basic elements of the fracture control plan are as follows.

1-Establishing of a formula suitable for determination of the stress intensity factor arising ahead of circumferentially cracked shaft loaded in torsion mode .

2-Identification of the parameters that may contribute to fracture of the member or component.

3-Determination of material fracture toughness to ensure the safety and reliability of the component against fracture

These three elements can be interrelated using the concepts of fracture mechanics to predict the susceptibility of a component fracture. [1-4] Furthermore. unstable crack propagation can be governed by the material toughness. the crack size and the stress level

From the fundamental consideration the torsional mode fracture toughness. of material used for manufacturing the rotating shafts is defined as the value of stress intensity factor. at which the crack extension starts [15].

1-ESTABLISHMENT OF THE MATHEMATICAL FORMULA

The stress field near an elastic crack tip can be characterized by the stress intensity factor Thus. the stress intensity factor for torsional mode loading of circumferentially cracked round bars can be represented by a new proposed equation as follows :

$$K_{III} = \frac{16T}{\sqrt{(\pi/\alpha)}\, d_c^{3/2}} \sqrt{\left(\frac{2a}{d_c}\right)} \left\{ 1 + 3\left(\frac{2a}{d_c}\right) + 3\left(\frac{2a}{d_c}\right)^2 + \left(\frac{2a}{d_c}\right)^3 \right\} \tag{1}$$

where

T	applied torque
d_c	cracked bar diameter
d_s	solid bar diameter
α	(d_c/d_s) ratio
a	circumferentially crack depth
K_{III}	torsional stress intensity factor

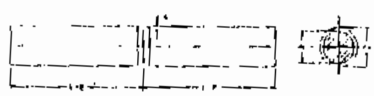

Fig. 1.Circumferentially cracked round bar

Details of the actual stress procedure and the derivation of equation (1) are indicated in Appendix-A

It is of practical interest to compare the proposed equation with the available equation in literature. which was given by Harris [5]

$$K_{III} = \frac{16T}{\sqrt{\pi}\, d_s^{3/2}} \sqrt{\left(\frac{2a}{d_s}\right)} \left[1 + \left(\frac{2a}{d_s}\right)\right]^{3/2} \left[1 + \frac{64}{9}\left(\frac{2a}{d_s}\right)\right]^{1/2} \tag{2}$$

Plotted curves are shown in Fig. 1 Comparison between the two relationships for K_{III} namely. equations (1) and (2) is discussed in section of results and discussion

In addition, it is also important to take into careful consideration the dependence of the torque-twist correlation on the diameter of cracked bar. Therefore, the following formula has been proposed:

$$\left(\frac{\theta_c}{T}\right) = \frac{32\,L}{G\,\pi\,d_c^4}\left[1+4\left(\frac{2a}{d_c}\right)+6\left(\frac{2a}{d_c}\right)^2+4\left(\frac{2a}{d_c}\right)^3+\left(\frac{2a}{d_c}\right)^4\right] \qquad (3)$$

Details are indicated in Appendix-B

where

θ_c twist angle of the cracked bar

G elastic modules of the rigidity

By using the proposed equation (3), the torque-twist relationship for circumferentially cracked round bars loaded in mode-III is obtained. Thereupon, the dependence of torque-twist correlation on the diameter of cracked bar plotted as shown in Fig. 2.

As would be discussed shortly, the amplitude of stress distribution designated by the stress intensity factor K_{III} is used to evaluate stress in a pre-cracked body [6,7]. Therefore, like in the case of an uncracked body, one can write a failure criterion of a pre-cracked body. Failure occurs if the value of stress intensity factor reached the value of fracture toughness of the same material as, [8-10]:

$$K_{III} = K_{IIIc} \qquad (4)$$

where K_{IIIc} is the critical value of K_{III} at which failure occur

The amplitude of stress intensity factor, K_{III} depends only on loading, size and geometry and is independent of the material resistance to failure.[3] On the other hand, if equation (4) is to effectively predict failure, K_{IIIc} should be independent of size, geometry and loading. It is then obvious that K_{III} is similar to stress and K_{IIIc} is similar to strength. [11].

In such cases, we need to use the fracture mechanics concepts to describe the mechanical environment at the crack tip [6-12]. The stress field near the crack tip can also be characterized by the normalized stress intensity factors. Thus, based on the proposed equation (1) the stress intensity factor for mode-III loading of circumferentially cracked round bars can be given in the dimensionless form as follows

$$\frac{K_{III}}{T}\,d_c^{5/2} = \frac{16}{\sqrt{(\pi/\alpha)}}\sqrt{\left(\frac{2a}{d_c}\right)}\left(1+3\left(\frac{2a}{d_c}\right)+3\left(\frac{2a}{d_c}\right)^2+\left(\frac{2a}{d_c}\right)^3\right) \qquad (5a)$$

$$\frac{K_{III}}{\theta_c}\,d_c^{-3/2} = \frac{G\sqrt{(\pi/\alpha)}}{2L}\sqrt{\left(\frac{2a}{d_c}\right)}\left(1+\left(\frac{2a}{d_c}\right)\right)^{-1} \qquad (5b)$$

Thereupon, the terms (K_{III}/T) and (K_{III}/θ_c) can be evaluated as given by equations (5a,5b). Hence, the flow curves are plotted as shown in Figs. 4 and 5.

Furthermore, the ratio between the torsional mode stress intensity factor K_{III} and the yield shear stress τ_y deduced from the following equation, required to be known for twist-torque relationship.

$$\left(\frac{K_{III}}{\tau_y}\right) = \frac{32\,L}{G\,\sqrt{(\pi/\alpha)}\,d_c^{3/2}}\left(\frac{T}{\theta}\right)\sqrt{\left(\frac{2a}{d_c}\right)}\left[1+4\left(\frac{2a}{d_c}\right)+6\left(\frac{2a}{d_c}\right)^2+4\left(\frac{2a}{d_c}\right)^3+\left(\frac{2a}{d_c}\right)^4\right] \qquad (6)$$

Plot of this relationship is given as shown in Fig. 6. Details are given in Appendix-C

2-IDENTIFICATION OF THE RELATED PARAMETERS

The stress intensity factor is necessarily a linear elastic parameter and therefore it is important to ensure that, the plastic zone at the crack tip is small as compared to other dimensions

In the circumferentially cracked bar, the shear stress may exceed the yield strength of the material at distance very close to the crack tip, thereby a plastic zone forms the crack tip. However, the plastic zone is small as compared to the specimen size, and the shear stress near the crack tip obeys a $\sqrt{1/r^*}$ type of singularity. The strain distribution is of $\sqrt{1/r^*}$ type near the crack tip, but away from the crack tip is of linear type. The strain distribution in the ligament of a circumferentially pre cracked round bar within the plastic zone is given by

$$\gamma = \gamma_c (\rho_z / r^*)^{1/2} \tag{7}$$

and the plastic zone size ρ_z is then:

$$\rho_z = (\gamma / \gamma_c)^2 r^* \tag{8}$$

where r* is the distance from the crack tip to the point of changes from linear to nonlinear.

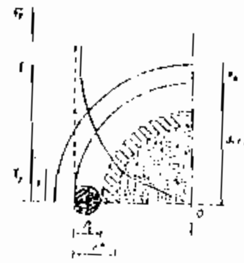

Fig.ıı Strain distribution change from linear to nonlinear.

It has been shown by Mc Clintook and Irwin [13] also given by Krafft and Sullivan [12] that the mode-III plastic zone at the crack tip of a crack is equal to an extension of the crack by an amount equal to $\rho_z/2$.

In reality, when the structure is loaded the plastic zone and the crack both grow simultaneously. Thus, the effective crack depth a_e is defined as a sum of initial crack depth a_o, and the crack growth δ, due to growth of plastic zone.

3-DETERMINATION OF FRACTURE TOUGHNESS

The contribution of the limited amount of crack growth to the deviation from linearity is also evaluated from the analysis of the R-curve data. R-curve is defined as a plot of crack growth resistance in a material as a function of actual or effective crack extension [14]. R-curves characterize the resistance to fracture of material during incremental slow-stable crack extension.

American society for testing of materials have proposed a tentative recommended practice for R-curve determination E561-76T [15]. Only a brief summary of this practice is given as the following

1-The method is based on the assumption that, during slow stable fracturing the developing crack growth resistance is equal to crack extension force.

2-This recommended practice covers the determination of resistance to fracturing of metallic materials by R-curve.

The outline of the procedure for the determination of K_{IIIc} is illustrated in Fig. 7. The steps involved in the measurements of K_{IIIc}, as per this approach, would be the following:

1- Prepare circumferentially notched round bar specimens from the material being tested, each specimens has a different diameter and different value of $(2a/d_r)$.

2- Fatigue pr-crack and load the specimen in torsion and obtain twist-torque test record wherein the twist angle is measured across intermediate points on the gauge length of the specimen.

3- Determine the T value based on the measured value of θ_c

4- Using the calibration function for mode-III, calculate K_R corresponding to the T value obtained in step 3

5- Locate this point in a plot of (K_R / τ_y) versus crack growth rate of $2a_e^{1/2}$ (where $2a_e$ is the effective crack length) , thereupon connect the located points to draw dashed lines.

6- The dashed line represents the increase in K_R with increasing torque and increasing crack length for three different initial crack lengths

7- Draw solid lines through the origin as shown in Fig. 7. The solid lines represent the variation in K_{III} with crack length, 2a, for constant torque. That is, for a given torque level and increasing crack length (2a), K_{III} will increase as given in the proposed equation (1)

8- The three point of tangency where $K_R = K_{III}$ represent points of instability, or critical stress intensity factor. However, the points of tangency give the critical value of stress intensity factor K_{IIIc}, which can be considered as the fracture toughness value for the material under test.

EXPERIMENT

Experimental investigation to study the fracture behavior of the circumferentially cracked bars, loaded in torsion mode have conducted following the recommended practice for R-curve determination proposed by the American society for testing of materials[15]

On all specimens, the circumferential notch was V- shaped with 60° angle. After notching the specimen with a lathe tool the notch root radius obtained in the specimen was further reduced with the help of a razor blade which were mounted on the tool post of the lathe through a special fixture.

The pre-cracked specimens were loaded in torsion mode up to different torque levels ranged from 84 to 2267 kg mm to obtain the torque-twist records. The twist angles measured in these experiments are rather small. The twist angle was measured by using instrumentation specially developed for this work, which consist of a clip gauge together with a special fixture as shown in Fig. 3. This instrumentation permits measurement of the twist angle across two intermediate points on the gauge length. Where the particular test specimen diameter is varied as 5, 10, 15and 20 mm.

The angular displacement at the crack tip on the crack plane, can be evaluated from measurement of the linear displacement between two points located across the crack plane and at the same radial distance from longitudinal axis of the circumferentially cracked specimen. The developed instrumentation for this purpose consists of a clip gauge and a sleeve assembly

RESULTS AND DISCUSSION

The stress intensity factor in torsion mode K_{III} were calculated in accordance with different specimen diameters using the calibration functions as given by the proposed equation (1) and equation (2) reported by Harris.[5] Resulted curves are plotted in the space of (K_{III}/τ_c) versus $(2a/d_c)$ values for different cracked round bar diameters as shown in Fig. 1

As can be see from this plot, the K_{III} calibration factors as given by proposed equation (1) and by Harris equation (2) are in acceptable agreement for $(2a/d_c)$ values less than 0.1. On other hand, the calibration function given by Harris is 20-73% lower than the exact values beyond 0.1 (in the range of $(2a/d_c)$ changed from 0.1 to 0.9). This observation is shown clearly in Fig. 1

In addition, if one were to consider the plastic flow curves of the specimens with different diameters and given (2a/dc) ratio all plastic flow curves shown in Figs.(4-6) would lie within a maximum band of ± 16%. The analytical results are located at the middle of the scatter band representing experimental data

The comparison of the experimental and analytical results shows a little deviation. The possible reason for this deviation could be the differences between the configuration of circumferential notch considered in the theoretical analysis and the one actually produced on the specimens for the experimental investigation

Another source of deviation could be the non-uniform, eccentric crack developed during the pre cracking stage. On the other hand, the shear modules values obtained experimentally are 28% lower than the shear modules values reported in the handbooks (Handbook value is 8×10^3 kg/mm^2 while the experimental value using machine instrumentation is 5.76×10^3 kg/mm^2) Moreover, the mechanical properties of steel used for experimental investigation are given as Tensile yield strength is 42 kg/mm^2, Ultimate tensile strength is 61 kg/mm^2 and Shear yield strength is 19 kg/mm^2)

One should also note that, the deviation in the results could be originated from the inherent variation in the properties of the material from one specimen to another. This will produce an erroneous results. All these factor added together can produce the deviation between the experimental and analytical results

Thereafter, the procedure for K_{IIIc} determination as described above, has been followed using the proposed equation (1) and also Harris equation (2). Thus, using equation (1) the K_{IIIc} value is indicated as the value associated with point of tangency between the solid line representing K_{III} curve and the dashed line of K_R representing R-curve itself as shown in Fig. 7. The solid lines represent the variation in K_{III} with crack length, 2a, for constant torque. The dashed lines represent the increase in K_R with increasing torque and increasing crack length for different initial crack lengths

The three point of tangency shown in Fig. 7, where $K_R = K_{IIIc}$ represent points of instability, or critical stress intensity factor. However, the points of tangency give the critical value of stress intensity factor K_{IIIc} =152 $MNm^{3/2}$ which can be considered as the fracture toughness value for the material under test. (where the fracture toughness values for steels ranged from 80 to 260 $MNm^{3/2}$ as reported in literature)

Furthermore, the plotted curves shown in Fig. 8., indicate that, the application of Harris equation gives K_R values 170 to 300 % higher than the analytical values of K_{III}. Therefore, no intersection occurs between the solid and dashed lines. However, Harris s equation would then be inconsistent with the basic requirement of determination of K_{IIIc} by R-curve procedure

Accordingly, if one were to take into account the above, the results indicated from the plastic flow curves shown in Figs. 4-6 and the material fracture toughness resulted from Fig. 7 can indeed be considered good. This results confirm the validity of the proposed equations.

CONCLUSION

1-A fracture-control plan for rotating shafts has been proposed in this investigation, to constitute the basis of a procedure for determining the torsion mode fracture toughness, K_c.

2-Considering the fracture mechanics principals, a new calibration formula has been developed for determination of the basic elements of a fracture-control plan

3-A procedure of elastic-plastic analysis using R-curve approach has been developed for determination of the fracture toughness in mode-III to control and prevent torsional mode fracture of rotating shafts

4-The results indicate that, the fracture toughness determined by using the proposed procedure is independent of size and geometry of the specimens.

5-The developed fracture control plan for the rotating shafts results in a better understanding of the possible fracture characteristics of the given application under consideration. This results confirm the validity of the proposed control plan

6-The analysis presented in this investigation presents an efficient method of fracture characterization of fracture behavior for rotating shafts subjected to torsional mode

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APPENDIX-A
FORMATION OF THE PROPOSED EQUATION (1) FOR DETERMINATION OF TORSION MODE STRESS INTENSITY FACTOR

A crack once formed, may move forward in a controlled manner, and the rate of crack growth depends upon a convenient crack tip describing parameter, namely the stress intensity factor denoted by K (SIF). The stress intensity factor can be determined by the familiar form of:

$$K \; \alpha \; \sigma_s \sqrt{\pi b} \qquad (i)$$

where σ_s is the applied stress, and b is the crack length

The basic stress considered here is due to a torque on the round bar, thus the value of torsion shear stress for solid and cracked bars of circular cross-section area are represented respectively by:

$$\tau_s = \frac{16T}{\pi d_s^3} \qquad (ii)$$

$$\tau_c = \frac{16T}{\pi d_c^3} \qquad (iii)$$

where d_s is the solid bar diameter, T is the external moment, τ_s is the applied shear stress for the solid bar, dc is the cracked bar diameter and τ_c is the torsion stress in a cracked circular bar.

Thus, for a circumferential crack loaded in the shearing mode, the stress intensity factor in the neighborhood of the crack tip is given by [Irwin] as

$$K_{III} = \tau_c \sqrt{\pi \, 2a} \qquad (iv)$$

where KIII is the mode-III stress intensity factor, 2a is the crack length and τ_c is the applied shear stress for the cracked bar loaded in mode-III.

In order to compare equation (ii) with equation (iii), it is necessary to write both equations in the same form, hence, equation (ii) can be given as the following

$$\tau_s = \frac{16T}{\pi(d_c + 2a)^3} = \frac{16T}{\pi d_c^3\left[1+(2a/d_c)\right]^3} \qquad (v)$$

Where the effective cracked bar diameter dc is given as

$d_c = d_s - 2a$

thus, $d = d_c + 2a$

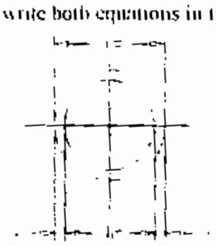


Fig.(a) the cross-section area of the circumferentially cracked round bar

Consequently one obtains after performing the necessary algebra

$$\tau_o = \frac{16T}{\pi d_c^3 \left[1 + 3\left(\frac{2a}{d}\right) + 3\left(\frac{2a}{d_c}\right)^2 + \left(\frac{2a}{d_c}\right)^3\right]}$$

Thus,

$$\tau_o \left[1 + 3\left(\frac{2a}{d_c}\right) + 3\left(\frac{2a}{d_c}\right)^2 + \left(\frac{2a}{d_c}\right)^3\right] = \frac{16T}{\pi d_c^3} = \tau_n \qquad \text{(vi)}$$

After the use of equation (iii), one can obtain:

$$\tau_n = \tau_o \left[1 + 3\left(\frac{2a}{d_c}\right) + 3\left(\frac{2a}{d_c}\right)^2 + \left(\frac{2a}{d_c}\right)^3\right] \qquad \text{(vii)}$$

Substituting equation (vii) into equation (iv) yields

$$K_{III} = \tau_n \sqrt{\pi 2a} \left[1 + 3\left(\frac{2a}{d}\right) + 3\left(\frac{2a}{d_c}\right)^2 + \left(\frac{2a}{d}\right)^3\right] \qquad \text{(viii)}$$

together with equation (ii), KIII can be given as the following:

$$K_{III} = \frac{16T}{\pi d_c^3} \sqrt{2\pi a} \left(1 + 3\left(\frac{2a}{d}\right) + 3\left(\frac{2a}{d_c}\right)^2 + \left(\frac{2a}{d_c}\right)^3\right) \qquad \text{(ix)}$$

Thus, the stress intensity factor for mode-III loading of circumferentially cracked round bar can be represented by a new proposed formula as follows :

$$K_{III} = \frac{16T}{\sqrt{(\pi/\alpha)}\, d_c^{5/2}} \sqrt{\left(\frac{2a}{d_c}\right)} \left(1 + 3\left(\frac{2a}{d_c}\right) + 3\left(\frac{2a}{d_c}\right)^2 + \left(\frac{2a}{d_c}\right)^3\right) \qquad \text{(I)}$$

where $\alpha = (d_c/d_o)$

From the above equation the crack shape and size parameter $f\left(\frac{2a}{d}\right)$ can be established as the follows:

$$f\left(\frac{2a}{d_c}\right) = 1 + 3\left(\frac{2a}{d_c}\right) + 3\left(\frac{2a}{d_c}\right)^2 + \left(\frac{2a}{d_c}\right)^3$$

APPENDIX-B
TORQUE-TWIST CORRELATION, EQUATION (3).

In a similar manner followed in appendix-A, the torque-twist correlation, therefore, required to consider the relation between the angular deflection in radians and torsional moment T for a round bar. Thus, the required equations for a solid bar and for a cracked round bar are given as :

$$\theta_o = \frac{TL}{G\,J} \quad \text{and} \quad \theta_c = \frac{32\,TL}{G\,\pi\,d_c^4}$$

$$\left(\frac{\theta_o}{T}\right) = \frac{32\,L}{G\,\pi\,d^4} \quad \text{and} \quad \left(\frac{\theta_c}{T}\right) = \frac{32\,L}{G\,\pi\,d_c^4} \qquad \text{(x)}$$

Moreover, the twist-torque ratio for a round circumferentially cracked bar loaded in torsion mode can be given as the following

$$\left(\frac{\theta_c}{T}\right)=\frac{32\,L}{G\,\pi\,d_c^4}=\frac{32\,L}{G\,\pi\,(d-2a)^4}\;\ldots$$

$$where\; d_c^4=(d_c-2a)^4$$

$$(d+2a)^4=d_c^4\left[1+4\left(\frac{2a}{d_c}\right)+6\left(\frac{2a}{d_c}\right)^2+4\left(\frac{2a}{d_c}\right)^3+\left(\frac{2a}{d_c}\right)^4\right]$$

$$\left(\frac{\theta_c}{T}\right)=\frac{32\,L}{G\,\pi\,d^4}\;\frac{1}{\left[1+4\left(\frac{2a}{d_c}\right)+6\left(\frac{2a}{d_c}\right)^2+4\left(\frac{2a}{d_c}\right)^3+\left(\frac{2a}{d_c}\right)^4\right]} \qquad \text{(ii)}$$

Comparison between equations (i) and (ii) lead to.

$$\left(\frac{\theta_c}{T}\right)=\left(\frac{\theta}{T}\right)\;\frac{1}{\left[1+4\left(\frac{2a}{dc}\right)+6\left(\frac{2a}{dc}\right)^2+4\left(\frac{2a}{dc}\right)^3+\left(\frac{2a}{dc}\right)^4\right]}$$

Thus

$$\left(\frac{\theta}{T}\right)=\frac{32\,L}{G\,\pi\,d_c^4}\left[1+4\left(\frac{2a}{d_c}\right)+6\left(\frac{2a}{d_c}\right)^2+4\left(\frac{2a}{d_c}\right)^3+\left(\frac{2a}{d_c}\right)^4\right] \qquad \textbf{(3)}$$

From the above equation the crack and shape and size parameter $f^*\left(\frac{2a}{d_c}\right)$ can be established as the following .

$$f^*\left(\frac{2a}{d_c}\right)=1+4\left(\frac{2a}{d_c}\right)+6\left(\frac{2a}{d_c}\right)^2+4\left(\frac{2a}{d_c}\right)^3+\left(\frac{2a}{d_c}\right)^4$$

APPENDIX-C

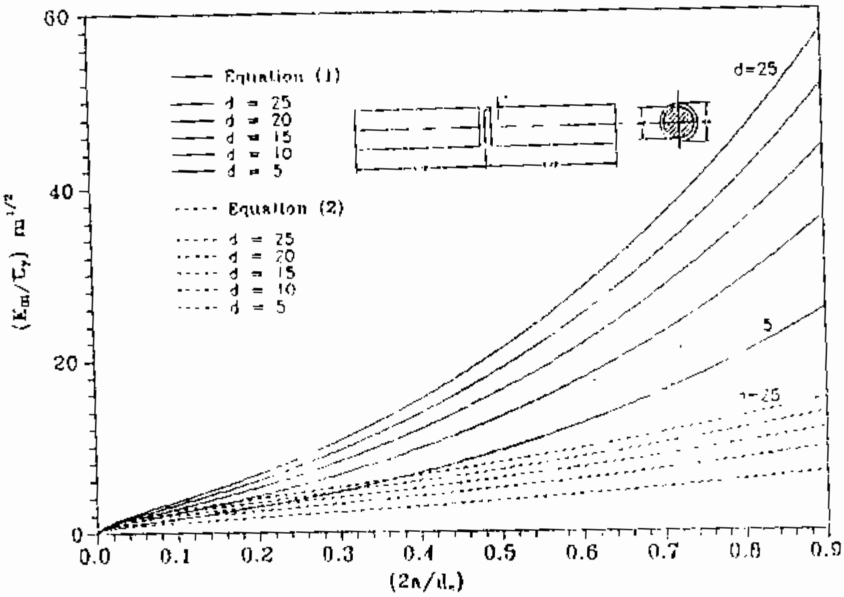
THE RELATION BETWEEN THE TORSIONAL MODE STRESS INTENSITY FACTOR K_{III} AND THE SHEAR YIELD STRESS τ_{ys} [EQUATION (6)]

The equation (3) in appendix-B may be expressed also as a function of torsional shear stress by substituting equation (3) in appendix-B into equation (iv)- appendix-A, one can obtain the following equation

$$\frac{K_{III}}{\tau_o}\left(\frac{\theta}{T}\right)=\sqrt{2\pi a}\left(\frac{32\,L}{G\,\pi\,d_c^4}\right)\left[1+4\left(\frac{2a}{d}\right)+6\left(\frac{2a}{d_c}\right)^2+4\left(\frac{2a}{d_c}\right)^3+\left(\frac{2a}{d_c}\right)^4\right]$$

Thereat the required stress equation is given as:

$$\left(\frac{K_{III}}{\tau_o}\right)=\frac{32\,L}{G\,\sqrt{(\pi/a)}\,d_c^4}\left(\frac{T}{\theta}\right)\sqrt{\left(\frac{2a}{d_c}\right)}\left[1+4\left(\frac{2a}{d}\right)+6\left(\frac{2a}{d_c}\right)^2+4\left(\frac{2a}{d_c}\right)^3+\left(\frac{2a}{d_c}\right)^4\right] \qquad \textbf{(6)}$$



Fig(1) Comparison of (K_{III}) calibration functions for the torsional mode loading of the circumferentially cracked round bars.

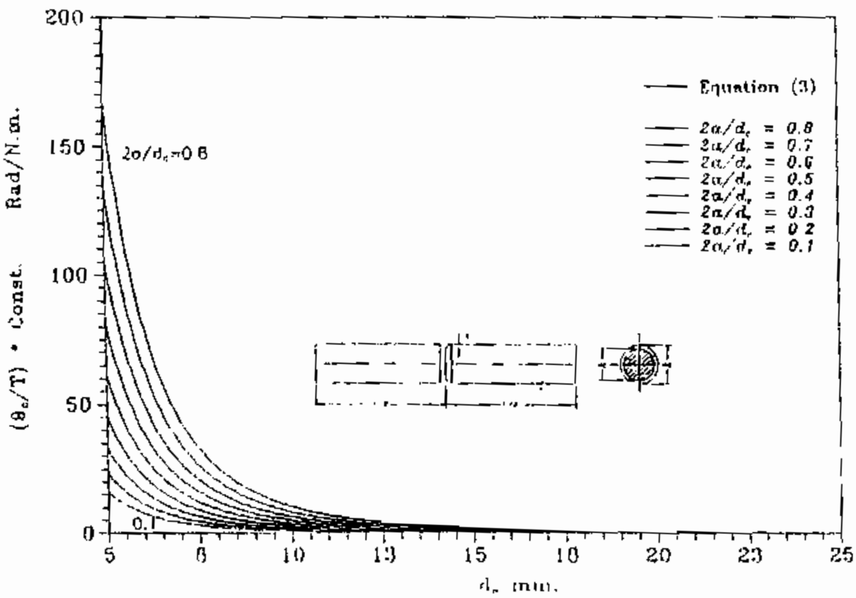


Fig (2) The dependence of torque-twist correlation on the diameter of the cracked bar.

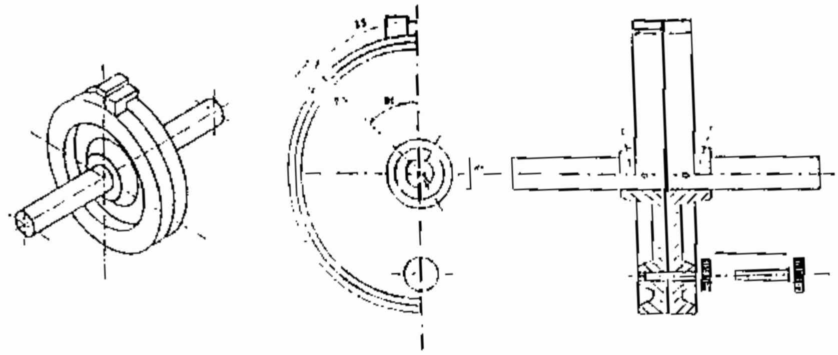


Fig 3 Specially designed fixture and clip gauge assembly for measurement of twist angle.

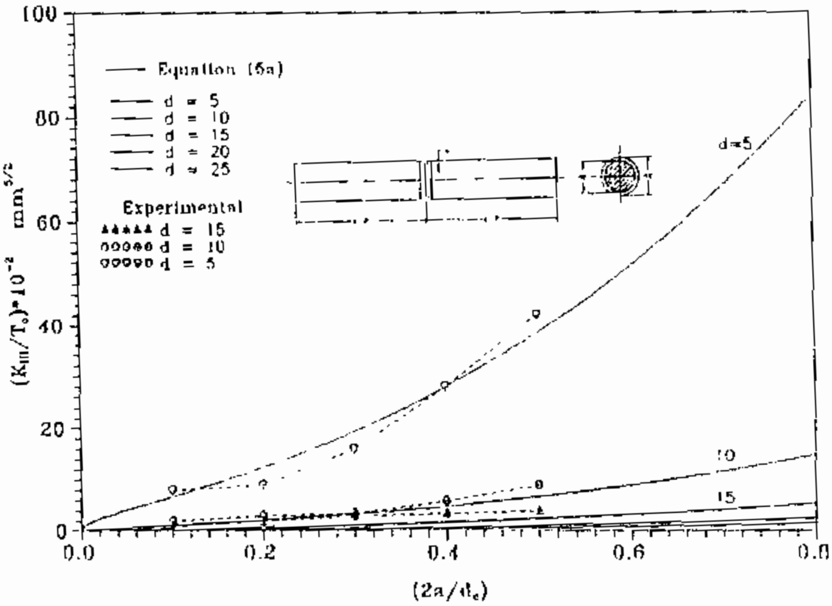
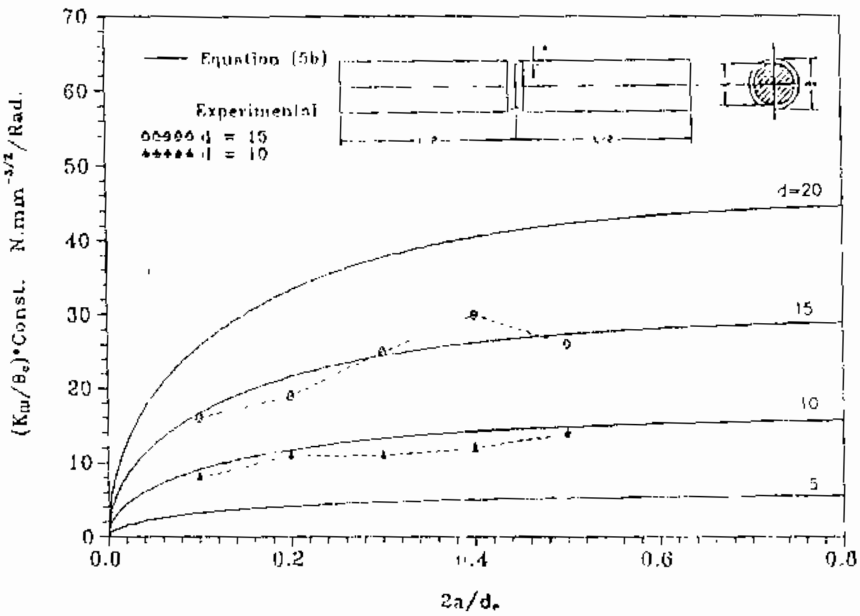


Fig. (4) Dependence of the normalized stress intensity factors on the crack depth ratio.



Fig(5) Effect of crack depth ratio $(2a/d_c)$ on (K_{III}/θ_c) for different specimen diameters.

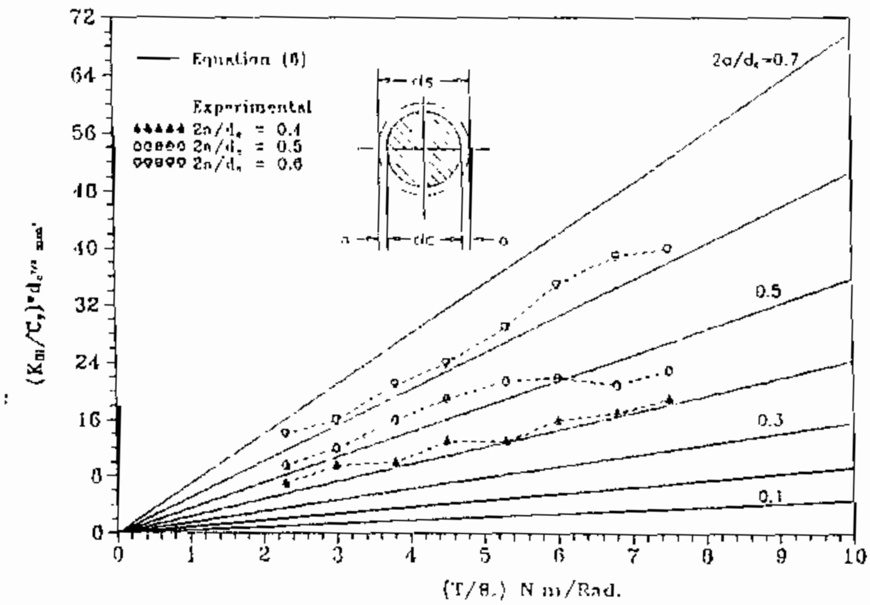


Fig. (6) Effect of crack depth ratio $(2a/d_c)$ on the variation of plastic flow with the torque-twist ratio

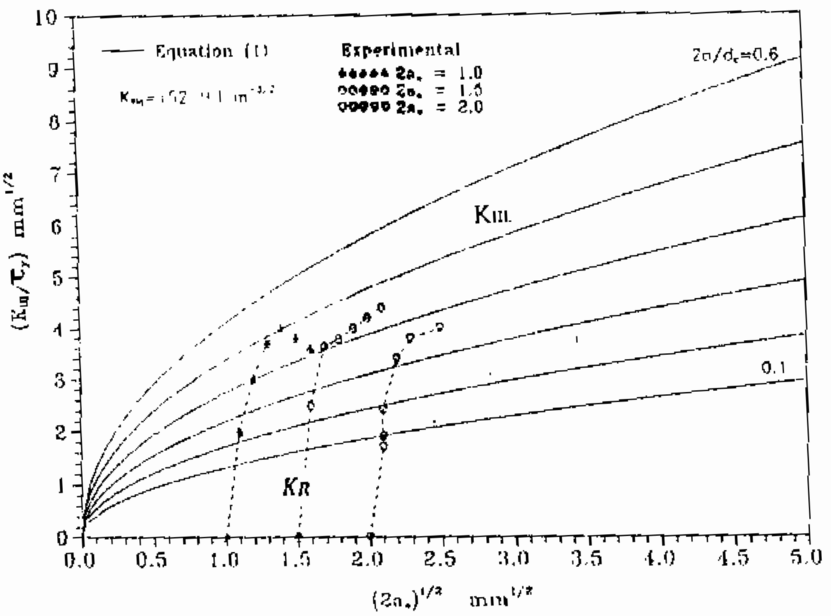
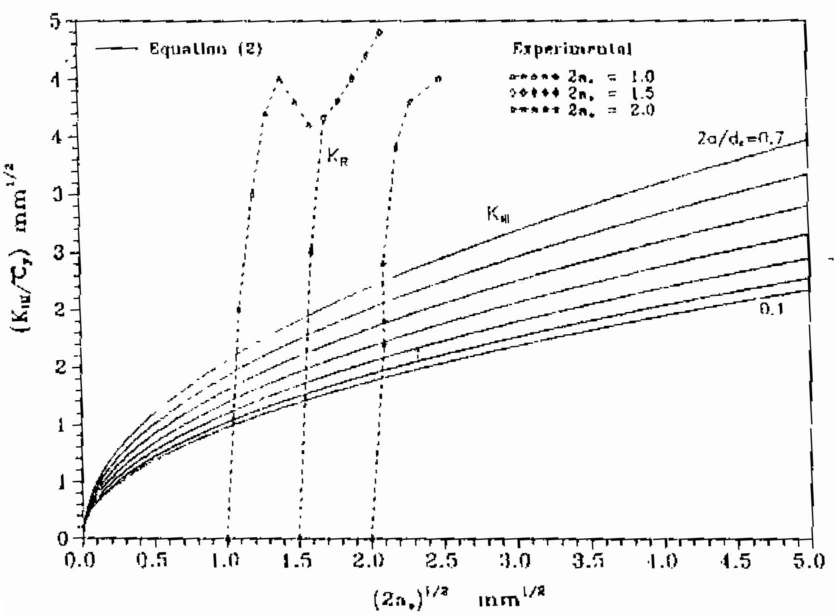


Fig.(7) Proposed procedure for determining K_{IIIc} under different conditions of initial crack length, a_0, using R-curve resistance curve analysis)



Fig(8) Application of Harris Equation for determining K_{III} under different conditions of initial crack length, a_0, using R-curve (resistance curve analysis)