



Answer all the following questions: [100 Marks]

Q1:

[50 Marks]

- (a) If the origin translated to the point $(1, -2)$ and the axes are rotated by an angle $\tan^{-1}(-0.5)$, find the new equation of the equation $14x^2 - 4xy + 11y^2 - 36x + 48y + 41 = 0$. (8Marks)
- (b) Prove that the equation $6x^2 + 7xy + 2y^2 - 11x - 7y + 3 = 0$ represents two straight lines and then find the point of intersection, the angle between them and the bisector equations. (8 Marks)
- (c) Find the vertex, focus, directrix and the latus rectum of the parabola $3x^2 + 12x - 8y = 0$, then find the equation of tangent at the point $(1, 4.5)$. (8 Marks)
- (d) Discuss and sketch the hyperbola $9x^2 - 4y^2 - 36x + 32y + 8 = 0$, then find the foci, directrices, asymptotes equations and the length of the latus rectum. (8 Marks)
- (e) Describe the locus of the point of intersection of two perpendicular tangents to the circle $x^2 + y^2 = r^2$. (8 Marks)
- (f) Find the common tangents drawn of the ellipses $\frac{x^2}{13} + \frac{y^2}{4} = 1$ and $\frac{x^2}{9} + \frac{y^2}{13} = 1$. (4 Marks)
- (g) Transform the following equations: (6 Marks)
- (i) $(x^2 + y^2)^2 = a^2(x^2 - y^2)$ to polar coordinates.
- (ii) $r = \frac{2a}{1 - \cos \theta}$ to cartesian coordinates and then classify it.

Q2:**[28 Marks]**

Find the following integrals:

(i) $\int x^4 (1+x^{5/2})^{1/2} dx$

(ii) $\int \frac{dx}{2+3 \tan x}$

(iii) $\int_0^{\pi/4} \ln(1+\tan \theta) d\theta$

(iv) $\int \frac{dx}{(x^2-6x+13)^2}$

(v) $\int \tan^3 x \sec^3 x dx$

(vi) $\int \frac{dx}{\sqrt{\sqrt{x}+1}}$

(vii) $\int \frac{x}{\sqrt{5x^2-4x}} dx$

Q3:**[22 Marks]**

- (a) Find the area bounded by the curves $x=1+y^2$ and $y=x-7$. (5 Marks)
- (b) Find the volume generated by revolving about the x-axis, the area bounded by the curves $x^2+y^2=25$, $3x-4y=0$ and $y=0$ lying in the first quadrant. (5 Marks)
- (c) Find the length of the curve $x^{2/3}+y^{2/3}=1$. If the curve is rotated about the x-axis in the first quadrant, then find the surface area of the solid generated. (7 Marks)
- (d) Applying Simpson's rule, obtain an approximate value of $\int_0^1 \frac{dx}{1+x^2}$, taking four equal intervals and hence obtain an approximate value of π , correct to four decimal places. (5 Marks)

Good Luck