

# Modeling of Synchronous Generators for Internal Faults Simulation Using MATLAB/SIMULINK

## نمذجة المولدات التزامنية لتمثيل ومحاكاة الأخطاء الداخلية باستخدام MATLAB/SIMULINK

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### ملخص البحث:

يعتبر المولد التزامني واحد من أهم عناصر الشبكة الكهربائية، وحدث عطل أو خطأ به قد يسبب حدوث عدم اتزان في نظام القوى الكهربائي، وبالتالي يؤدي إلى انقطاع القدرة الكهربائية للمصدر. لذلك فإن اكتشاف الأخطاء الداخلية في ملفات العضو الثابت يعتبر من أهم وظائف الحماية للمولد.

هذا البحث يقدم نموذج رياضي للمولد التزامني يمكن استخدامه في محاكاة الأخطاء الداخلية التي تحدث في ملفات العضو الثابت. هذا النموذج الرياضي يقوم بحساب التيارات الداخلة للمولد والخارجة منه وكذلك الجهود عند حدوث أخطاء داخلية في ملفات العضو الثابت، والنموذج المقترح قادر على محاكاة كل أنواع الأخطاء الداخلية في ملفات العضو الثابت، وتم تمثيل النموذج الرياضي باستخدام بيئة الـ MATLAB/SIMULINK. واثبت النموذج المقترح دقة وسهولة استخدام الماتلاب في تمثيل الآلات المعقدة، وذلك يجعله أكثر موثوقية عند استخدامه في أنظمة الحماية للمولد.

### Abstract

This paper presents a mathematical model for synchronous generator that can be used for internal faults simulation. The model can be used to produce realistic test waveforms for the evaluation of protection system used for synchronous generators. The model is capable of simulating any kind of internal faults on any type of winding configurations. The mathematical model is implemented in MATLAB-SIMULINK. It has been demonstrated that the MATLAB is a powerful tool to implement the complex machine model. The accuracy and simplicity of the model make it reliable and efficient in synchronous generator internal faults study.

**Keywords:** *Internal faults, Synchronous generator, Modelling, Dynamic simulation, Winding distribution.*

### 1. Introduction

The synchronous generator is one of the most important equipment in the electric power system. The maloperation of a generator causes the system to become unstable leading to possible supply interruption. Abnormal conditions can occur inside the generator due to faults in external system to which it is connected

and also due to faults inside the generator itself.

Detection of internal faults in the stator windings is one of the areas of concern for better protection of synchronous generators. The internal fault is defined as turn-to-turn or turn-to-frame insulation failure. It is needed to provide adequate protection for the synchronous generator to minimize the effects of the internal faults. These effects

include sever damage to the windings and possibly to the shaft and the coupling of the machine. A suitable synchronous generator model is required to enhance its protection. The internal fault models allow a detailed analysis taking into account the particular design, fault types and the location of fault.

The popular d-q axis model of the synchronous machine cannot be used in internal faults studies as it is derived on the assumption that the three-phase machine windings are evenly distributed and symmetrically displaced from one another, which is not the case of internal faults [1]. Whoever [2] presents a method for modelling and simulation of internal single phase-to-ground faults in d-q axis model in stator windings of large synchronous machines but the obtained results are very approximate compared to finite elements results. The synchronous machine internal faults are generally derived in the phase domain (abc model). There are many machine models available for internal fault analysis [3-9]. The method used in [3, 4] is the symmetrical component method. This model considers only the fundamental and the third harmonic components of the resultant fault signals and neglects the higher harmonics order. This leads to errors since internal faults give rise to higher harmonics order. The method used in [4, 5] does not consider the winding arrangement inside the machine and hence it is limited in application. The method described in [6-9] considers sinusoidally distributed windings. Such windings are also limited in applications. Some additional work on synchronous machines was done on the modelling of a salient-pole synchronous machine under dynamic eccentricity [10].

The multi-loop analysis method was proposed in [11], and has been widely used in [12-17]. The method considers the electrical machine as formed of several electric circuits, each composed of the actual loops that are formed by the coils.

The inaccuracy of this method was involved with the calculation of loop inductances. A voltage-behind-reactance (VBR) synchronous generator model [18] has been used for internal fault simulation in synchronous machine. The VBR model is attractive for use in large-scale multimachine systems and in real-time applications but on the cost of higher model complexity.

This paper presents important aspects of synchronous generator model. This model is capable of simulating normal operation and various types of external faults. The mathematical expression is used to relate the current and inductance, which are used as the variables in the simulation model. The paper also describes a suitable mathematical model for simulating internal faults in the synchronous generator. This model has been implemented in MATLAB /SIMULINK. The simulations are performed under various conditions: turn to ground fault, turn-to-turn (phase a to phase b) fault, three-phase fault, and turn-to-turn to ground (phase a to phase b to ground) fault.

## 2. Development of Generator Model for Internal Faults Analysis

A schematic representation of a synchronous generator with two damper coils is shown in Fig. (1). The windings subject to an internal fault are split into two parts with connection point available for insertion of fault branches.

In Fig. (1), each winding of the three-phase windings of the generator has been divided into parts, and it has three nodes. For each connection added inside the windings, a new winding is in fact added to the set which is mutually coupled with other windings.

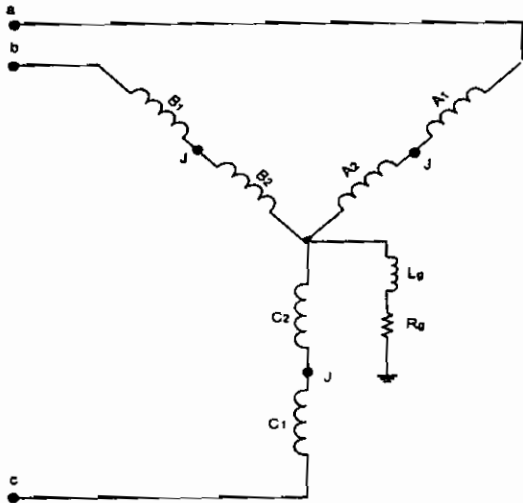


Fig.1 Circuit diagram of phase domain model of the synchronous generator

Consider sinusoidally distributed winding that has been used earlier in [6-8]. The winding distribution of phase winding a can be expressed as:

$$N_{ac}(\theta) = -N_s \sin(\theta) \tag{1}$$

Where  $N_{ac}$  is the actual turns of phase a winding,  $N_s$  is the number of effective turns and  $\theta$  is the angular position of rotor. This distribution is shown in Fig. (2).

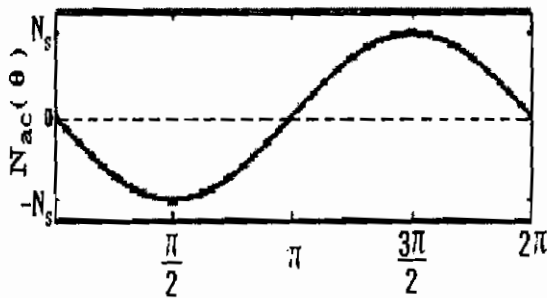


Fig.2 Sinusoidal approximation of phase "a" winding distribution.

If the point of fault splits the winding of phase a into sub-winding A1 with number of turns  $N1$  and sub-winding A2 with turns  $N2$ . The new winding distribution for these sub-winding are shown in Fig. 3, which are drawn with the assumption of a split at angle  $\alpha$ . According to Fig. (3) the sub-winding A1 has turns from 0 to  $\alpha$ , and the sub-winding A2 has turns from  $\alpha$  to  $\pi$ . The magnetic motive force (MMF) distribution is found by using Ampere's Law. If only

winding of phase a is energized with current  $i_a$ , the MMF can be expressed as:

$$MMF_a(\theta) = \int_{\theta-\pi}^{\theta} (i_a \cdot N_a(\theta)) d\theta \tag{2}$$

and the actual numbers of the sub-windings are expressed by:

$$N_{ac1} = \int_0^{\alpha} N_{ac}(\theta) d\theta \tag{3}$$

$$N_{ac2} = \int_{\alpha}^{\pi} N_{ac}(\theta) d\theta \tag{4}$$

The above section considered a sinusoidal distribution before fault application. However the approach is applicable to any arbitrary winding distribution by replacing plot in Fig. (2) and Fig. (3), this change the approach as it merely substitutes a new function instead of the sinusoidal function in Equation (1), this function is then used in equations (2, 3, and 4).

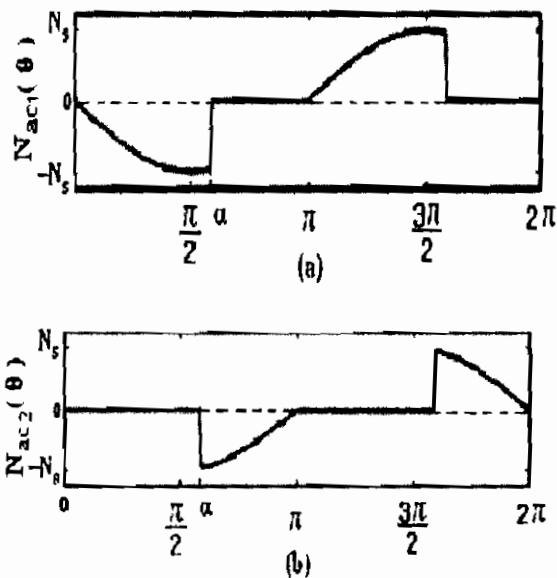


Fig.3 Functions of winding distribution for sub-windings "A1" and "A2".

### 2.1 Calculation of the Stator Inductance

For large synchronous generators, machine geometrical parameters and diagrams of rotor windings distribution are not readily available. However the machine electrical parameters are available and can

be used to simplify the inductance calculation.

Consider the self inductance of winding  $A1$  and the mutual inductance between winding  $A1$  and  $A2$ . Winding  $A1$  has  $N1$  effective turns with a magnetic axis angle of  $\varphi_1$  relative to phase a magnetic axis. Winding  $A2$  has  $N2$  effective turns with a magnetic axis angle of  $\varphi_2$  relative to the phase a magnetic axis.

### 2.1.1 The self inductance of stator winding

The magnetic flux acting on the d-axis due to a current in winding  $A1$  is given by:

$$\phi_{da1} = (N_{a1} i_{a1} (P_d \cos(\theta - \varphi_1))) \quad (5)$$

In similar manner, the magnetic flux acting on the q-axis is given by:

$$\phi_{qa1} = (N_{a1} i_{a1} P_q \sin(\theta - \varphi_1)) \quad (6)$$

Where:  $P_d$  is the d-axis winding permeance and  $P_q$  is the q-axis winding permeance. The flux linkage of winding  $A1$  is given by:

$$\lambda_{a11} = (\lambda_{a11})_l + N_{a1} (\phi_{da1} \cos(\theta - \varphi_1) - \phi_{qa1} \sin(\theta - \varphi_1)) \quad (7)$$

where:  $(\lambda_{a11})_l$  is the flux linkage due to leakage flux and given by:

$$(\lambda_{a11})_l = N_{lac} P_{l1} i_{a1} \quad (8)$$

where  $P_{l1}$  is the permeability of the leakage path. Substituting with equations (5, 6 and 8) into equation (7) gives:

$$\lambda_{a11} = N_{lac} P_{l1} i_{a1} + N_{a1}^2 i_{a1} P_o + N_{a1}^2 i_{a1} P_2 \cos(2(\theta - \varphi_1)) \quad (9)$$

Where:

$$P_o = \left( \frac{P_d + P_q}{2} \right)$$

$$P_2 = \left( \frac{P_d - P_q}{2} \right)$$

The self inductance of the winding  $A1$  is:

$$L_{a11} = N_{lac} P_{l1} + N_{a1}^2 P_o + N_{a1}^2 P_2 \cos(2(\theta - \varphi_1)) \quad (10)$$

### 2.1.2 The mutual inductance of the stator windings

Consider the mutual inductance between  $A1$  and  $A2$  windings. The flux

linkage to winding  $A2$  due to a current in winding  $A1$  is given by:

$$\lambda_{a21} = N_{a2} ((\phi_{da1} \cos(\theta - \varphi_{a2})) - (\phi_{qa1} \sin(\theta - \varphi_{a2}))) \quad (11)$$

Substituting from equation (5 and 6) into equation (11) gives:

$$\lambda_{a21} = N_{a1} N_{a2} i_{a1} (P_o \cos(\varphi_{a1} - \varphi_{a2}) + P_2 \cos(2(\theta - \frac{\varphi_{a1} + \varphi_{a2}}{2}))) \quad (12)$$

The mutual inductance  $L_{a12}$  between winding  $A1$  and  $A2$  is given by the ratio between the flux  $\lambda_{a12}$  and current  $i_{a1}$ .

$$L_{a12} = L_{a21} = N_{a1} N_{a2} P_o \cos(-\varphi_{a1} + \varphi_{a2}) + N_{a1} N_{a2} P_2 \cos(2(\theta - \frac{\varphi_{a1} + \varphi_{a2}}{2})) \quad (13)$$

The general expression of the self and mutual inductance of the stator is given by:

$$L_{pjl}(\theta) = \begin{cases} N_{poc} P_{pl} + N_p^2 P_o + N_p^2 P_2 \cos 2(\theta - \varphi_p) & i = j \\ N_p N_{pj} P_o \cos(\varphi_p + \beta_p - (\varphi_{pj} + \beta_p)) + N_p N_{pj} P_2 \cos 2(\theta - \frac{(\varphi_p + \beta_p) - (\varphi_{pj} + \beta_p)}{2}) & i \neq j \end{cases} \quad (14)$$

where:

$$\beta_p = \begin{cases} 0 & p = \text{phase}(a) \\ \frac{2\pi}{3} & p = \text{phase}(b) \\ -\frac{2\pi}{3} & p = \text{phase}(c) \end{cases}$$

## 2.2 The Mutual Inductance between the Stator and the Rotor Windings

Our goal is to investigate the internal faults in the stator windings, the windings distribution and electrical parameters of the rotor windings are not affected by the internal faults in the stator windings. However the mutual inductances between the rotor and stator should be recalculated since the winding distributions of the faulted windings have changed (for example the mutual inductance between field winding ( $f$ ) and winding ( $A1$ )).

The flux linkage to the field winding due to a current in winding  $A1$  is given by:

$$\lambda_{fa1} = N_f \phi_{da1} \quad (15)$$

By substituting from equation (5)

$$\lambda_{fa1} = N_{a1} N_f i_{a1} P_d \cos(\theta - \varphi_{a1}) \quad (16)$$

The inductance is expressed as:

$$L_{fa1} = N_{a1} N_f P_d \cos(\theta - \varphi_{a1}) \quad (17)$$

The general expression of mutual inductance between the stator and the rotor is given by:

$$L_{np}(\theta) = \begin{cases} N_{pf} N_R P_d \cos\theta - (\varphi_{pf} + \beta_p) & R=f \text{ or } D \\ N_{pq} N_R P_q \cos\theta - (\varphi_{pq} + \beta_p) & R=Q \end{cases} \quad (18)$$

Where  $D$  represents the d-axis damper winding and  $Q$  is the q-axis damper winding.

### 2.3 The Self and Mutual Inductance of the Rotor

The self and mutual inductance of the rotor is not affected by the splitting of windings of the stator.

## 3. Mathematical Model of the Synchronous Machine

The electromechanical behavior of a synchronous machine is described by the differential equations. These differential equations are developed to handle internal faults in stator windings. The voltage and current conditions for each type of internal faults are illustrated in table 1.

Table.1 The boundary conditions of different types of internal faults

Fault Type	Voltage Conditions	Current Conditions
Phase a to ground fault	$v_{a2} + v_n = 0$ $v_{a1} + v_{a2} = v_{an}$	$i_{a1} = i_a$ $i_{b1} = i_{b2}$ $i_{c1} = i_{c2}$
Phase a to b fault	$v_{a2} = v_{b2}$	$i_{a1} = i_a$ $i_{b1} = i_b$ $i_{c1} = i_{c2}$
Phase a and b to ground fault	$v_{a2} + v_n = 0$ $v_{b2} + v_n = 0$	$i_{a1} = i_a$ $i_{b1} = i_b$ $i_{c1} = i_{c2}$
Phase a, b and c to ground fault	$v_{a2} + v_{an} = 0$ $v_{b2} + v_{bn} = 0$ $v_{c2} + v_{cn} = 0$	$i_{a1} = i_a$ $i_{b1} = i_b$ $i_{c1} = i_c$

The instantaneous values of voltage in the independent windings of the synchronous machine can be given by:

$$[v] = [R].[i] + \frac{d}{dt}[\lambda] \quad (19)$$

where:

$$[v] = [v_{a1} v_{a2} v_{b1} v_{b2} v_{c1} v_{c2} v_f 0 0]^T$$

$$[R] = \text{diag}[R_{a1} R_{a2} R_{b1} R_{b2} R_{c1} R_{c2} R_f R_D R_Q]$$

$$[i] = [i_{a1} i_{a2} i_{b1} i_{b2} i_{c1} i_{c2} i_f i_D i_Q]^T$$

$$[\lambda] = [\lambda_{a1} \lambda_{a2} \lambda_{b1} \lambda_{b2} \lambda_{c1} \lambda_{c2} \lambda_f \lambda_D \lambda_Q]^T$$

### 3.1 Internal Single Line to Ground Fault Equations

Fig. (4) illustrates the stator of synchronous machine for single line to ground fault at winding of phase a.

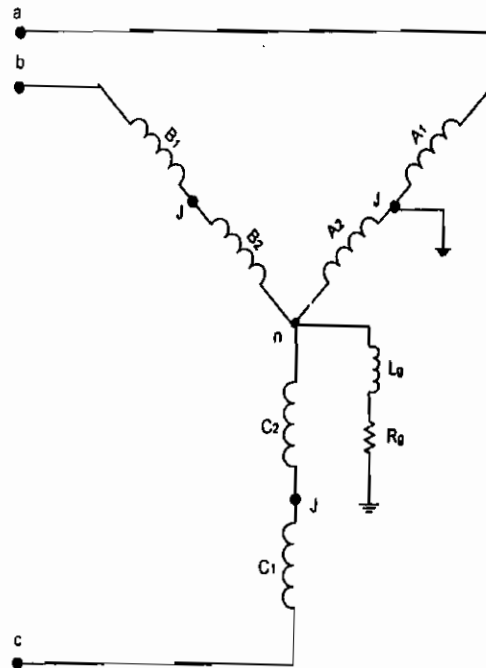


Fig. 4 Synchronous machine representation during an internal phase a to ground fault

The flux linkages during the fault are:

$$[\lambda_1] = [L_1(\theta)]i_1 \quad (20)$$

Where:

$$L_1(\theta) = \begin{bmatrix} L_{b11} & L_{b1b2} & L_{b1b1} + L_{b1b2} \\ L_{b1b2} & L_{b2b2} & L_{b2b1} + L_{b2b2} \\ L_{b1b1} + L_{b1b2} & L_{b2b1} + L_{b2b2} & (L_{b11} + 2L_{b1b2} + L_{b22}) \\ L_{b1f} & L_{b2f} & L_{b1f} + L_{b2f} \\ L_{b1D} & L_{b2D} & L_{b1D} + L_{b2D} \\ L_{b1Q} & L_{b2Q} & L_{b1Q} + L_{b2Q} \end{bmatrix}$$

$$\begin{bmatrix} L_{b1c1} + L_{b1c2} & L_{b1f} & L_{b1D} & L_{b1Q} \\ L_{b2c1} + L_{b2c2} & L_{b2f} & L_{b2D} & L_{b2Q} \\ (L_{b1c1} + L_{b1c2} + L_{b2c1} + L_{b2c2}) & L_{b1f} + L_{b2f} & L_{b1D} + L_{b2D} & L_{b1Q} + L_{b2Q} \\ (L_{c11} + 2L_{c1c2} + L_{c22}) & L_{c1f} + L_{c2f} & L_{c1D} + L_{c2D} & L_{c1Q} + L_{c2Q} \\ L_{c1f} + L_{c2f} & L_f & L_{fD} & L_{fQ} \\ L_{c1D} + L_{c2D} & L_{Df} & L_D & L_{DQ} \\ L_{c1Q} + L_{c2Q} & L_{Qf} & L_{QD} & L_Q \end{bmatrix}$$

$$L_2(\theta) = \begin{bmatrix} L_{b11} & L_{b1b2} & L_{b1b1} & L_{b1b2} \\ L_{b1b2} & L_{b22} & L_{b2b1} & L_{b2b2} \\ L_{b1b1} & L_{b2b1} & L_{b11} & L_{b2b1} \\ L_{b1b2} & L_{b2b2} & L_{b1b2} & L_{b22} \\ L_{b1c1} + L_{b1c2} & L_{b2c1} + L_{b2c2} & L_{b1c1} + L_{b1c2} & L_{b2c1} + L_{b2c2} \\ L_{b1f} & L_{b2f} & L_{b1f} & L_{b2f} \\ L_{b1D} & L_{b2D} & L_{b1D} & L_{b2D} \\ L_{b1Q} & L_{b2Q} & L_{b1Q} & L_{b2Q} \end{bmatrix}$$

$$\begin{bmatrix} L_{b1c1} + L_{b1c2} & L_{b1f} & L_{b1D} & L_{b1Q} \\ L_{b2c1} + L_{b2c2} & L_{b2f} & L_{b2D} & L_{b2Q} \\ L_{b1c1} + L_{b1c2} & L_{b1f} & L_{b1D} & L_{b1Q} \\ L_{b2c1} + L_{b2c2} & L_{b2f} & L_{b2D} & L_{b2Q} \\ (L_{c11} + 2L_{c1c2} + L_{c22}) & L_{c1f} + L_{c2f} & L_{c1D} + L_{c2D} & L_{c1Q} + L_{c2Q} \\ L_{c1f} + L_{c2f} & L_f & L_{fD} & L_{fQ} \\ L_{c1D} + L_{c2D} & L_{Df} & L_D & L_{DQ} \\ L_{c1Q} + L_{c2Q} & L_{Qf} & L_{QD} & L_Q \end{bmatrix}$$

### 3.2 Internal Double-Line to Ground

#### Fault Equations

Fig. (5) illustrates the stator of synchronous machine under double-phase to ground fault at winding of phases a and b.

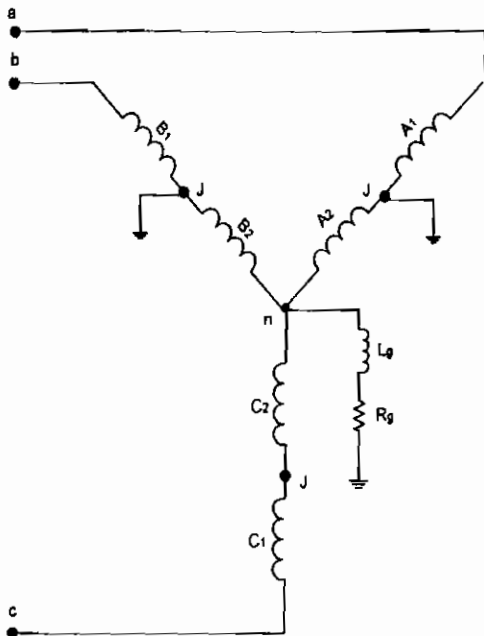


Fig. 5 Synchronous machine representation during an internal phase a to b to ground fault

The flux linkage during the fault is:

$$[\lambda_2] = [L_2(\theta)] [i_2] \tag{21}$$

where:

### 3.3 Internal Three-Phase to Ground

#### Fault Equations

Fig. (6) illustrates the stator of synchronous machine under three-phase to ground fault at winding of phase a, b, and c.

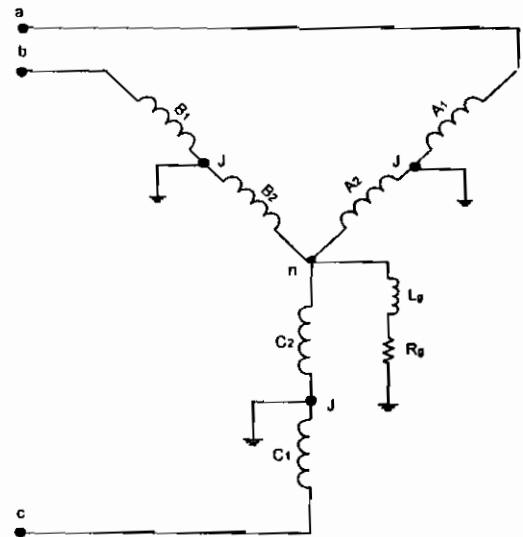


Fig. 6 Synchronous machine representation during an internal three phase to ground fault

The flux linkages during the fault are:

$$[\lambda_3] = [L_3(\theta)] [i_3] \tag{22}$$

where:

$$L_3(\theta) = \begin{bmatrix} L_{a11} & L_{a1a2} & L_{a1b1} & L_{a1b2} & L_{a1c1} \\ L_{a12} & L_{a22} & L_{a2b1} & L_{a2b2} & L_{a2c1} \\ L_{a1b1} & L_{a2b1} & L_{b11} & L_{b1b2} & L_{b1c1} \\ L_{a1b2} & L_{a2b2} & L_{b1b2} & L_{b22} & L_{b2c1} \\ L_{a1c1} & L_{a2c1} & L_{b1c1} & L_{b2c1} & L_{c11} \\ L_{a1c2} & L_{a2c2} & L_{b1c2} & L_{b2c2} & L_{c1c2} \\ L_{a1f} & L_{a2f} & L_{b1f} & L_{b2f} & L_{c1f} \\ L_{a1D} & L_{a2D} & L_{b1D} & L_{b2D} & L_{c1D} \\ L_{a1Q} & L_{a2Q} & L_{b1Q} & L_{b2Q} & L_{c1Q} \end{bmatrix}$$

$$\begin{bmatrix} L_{a1c2} & L_{a1f} & L_{a1D} & L_{a1Q} \\ L_{a2c2} & L_{a2f} & L_{a2D} & L_{a2Q} \\ L_{b1c2} & L_{b1f} & L_{b1D} & L_{b1Q} \\ L_{b2c2} & L_{b2f} & L_{b2D} & L_{b2Q} \\ L_{c1c2} & L_{c1f} & L_{c1D} & L_{c1Q} \\ L_{c22} & L_{c2f} & L_{c2D} & L_{c2Q} \\ L_{c2f} & L_f & L_{fD} & L_{fQ} \\ L_{c2D} & L_{Df} & L_D & L_{DQ} \\ L_{c2Q} & L_{Qf} & L_{QD} & L_Q \end{bmatrix}$$

**3.4 Rotor Motion Equations**

The electromechanical equation for a synchronous machine [20] follows directly from equating the inertia torque equal to the moment of inertia J times of the angular acceleration to the net mechanical and electric torque acting on the rotor.

$$J \frac{d^2\theta}{dt^2} = T_{mech} - T_{elec} \tag{23}$$

Where  $T_{mech}$  is the mechanical torque applied to rotor and  $T_{elec}$  is the electric torque and can be determined from the equation:

$$T_{elec} = \frac{1}{3} \left( \frac{1}{2} [\bar{i}_s] \frac{dL_{ss}}{d\theta} [\bar{i}_s] + [\bar{i}_s] \frac{dL_{SR}}{d\theta} [\bar{i}_R] \right) \tag{24}$$

Where  $L_{ss}$  is the self inductance of stator windings,  $L_{SR}$  is the mutual inductance between stator and rotor windings,  $i_R$  is the rotor currents and  $i_s$  is the stator currents.

**4. Modeling of Internal Faults in MATLAB/ SIMULINK**

Currently MATLAB/SIMULINK is a widely used simulation tool for dynamic systems, and a wide range of components will be involved for modelling large dynamic systems [19]. The system consists of three-phase synchronous generator connected to an infinite bus through a transmission lines. The circuit diagram of the modeled power system is shown in Fig. (7).

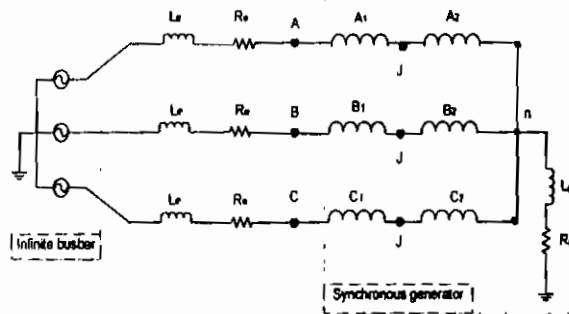
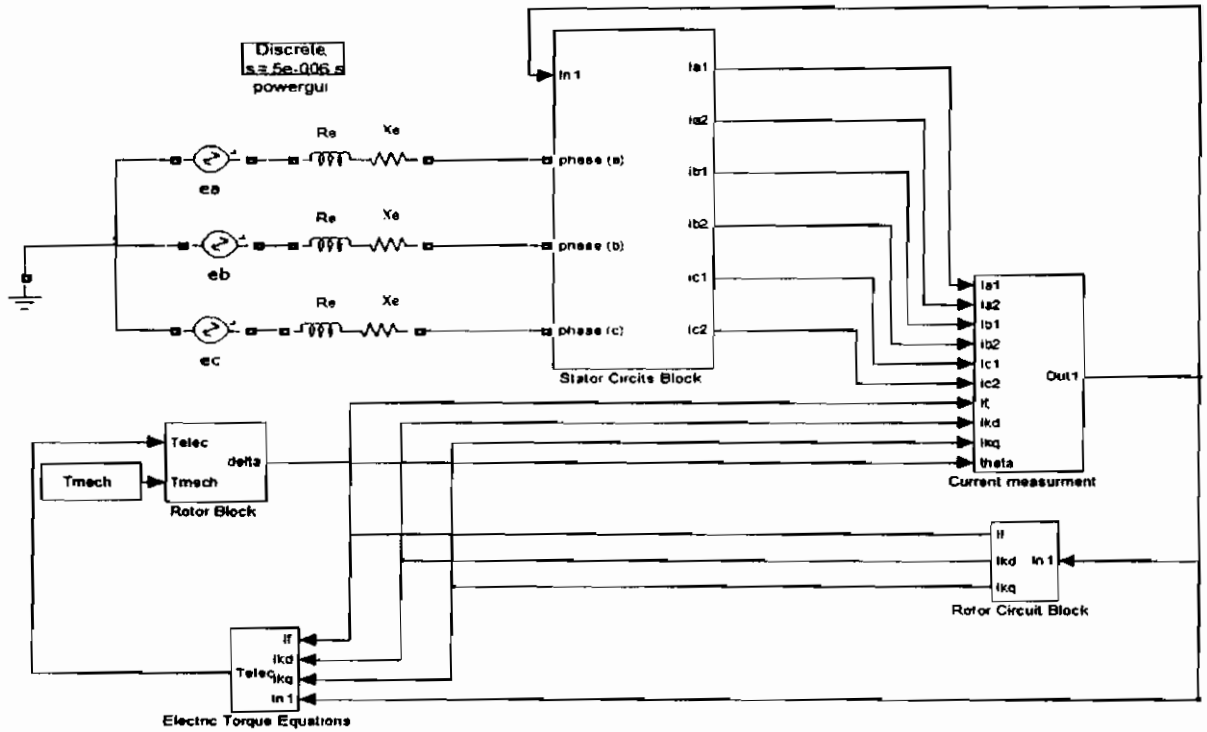
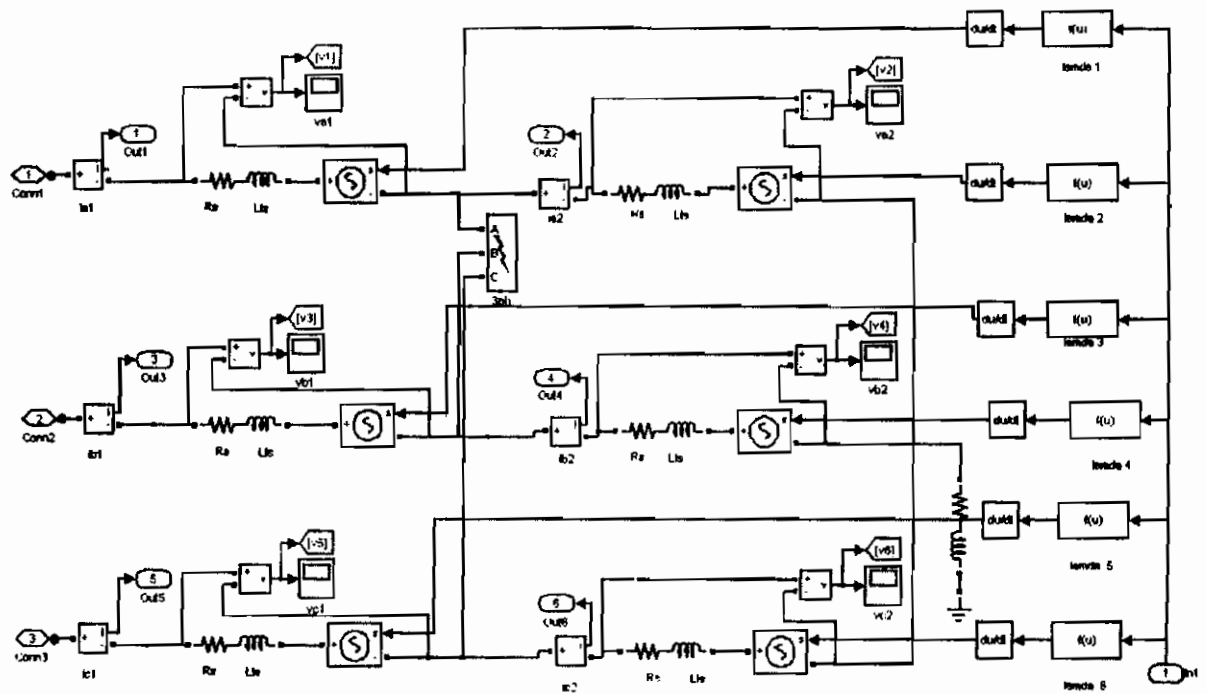


Fig. 7 Circuit diagram of the simulated model

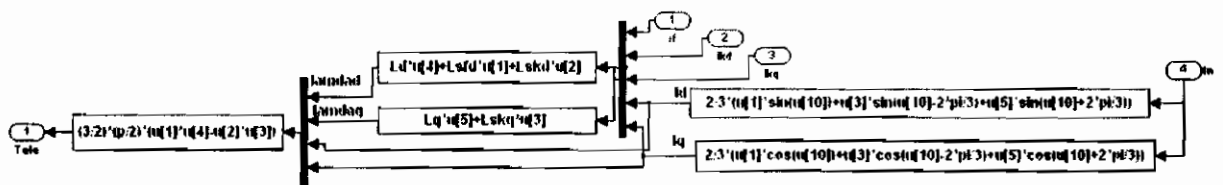
The model discussed above has been implemented in MATLAB/SIMULINK environment. Equation (19) is used as the basic differential equation for the implemented model. The MATLAB numerical integration allows all elements of the state vector and electrical torque to be computed, so we recommend using of the Dormand-Prince method (ode45). In order to simplify the implementation, the main computation part has been written in a function format. The fault location is specified in percentage of phase winding. For example a 40% ground fault on phase a means that 40% of the phase winding turns are between neutral and the fault point. The simulation model is shown in Fig. (8), where MATLAB function is used. The overall diagram of synchronous generator simulation is illustrated in Fig.(8a). It consists of four subsystems, stator winding circuit block, rotor winding circuit block, electric torque equations block, and rotor motion block.



(a) Overall diagram of synchronous generator simulation

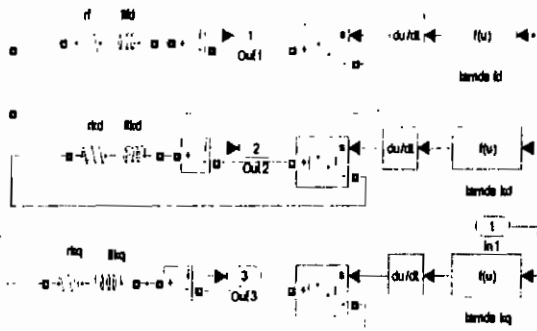


(b) Stator winding circuit model

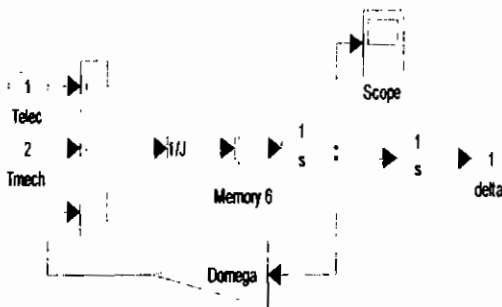


(c) Electric torque equations modeling





(d) Rotor winding circuit model



(e) Rotor model

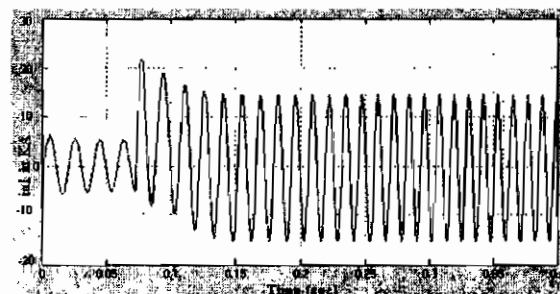
Fig.8 Synchronous machine simulation model with MATLAB/SIMULINK

### 5. Simulation and Results

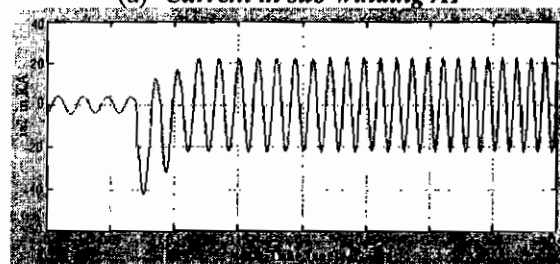
A synchronous machine system has been simulated with the developed abc direct phase model, the parameters of the synchronous machine was selected as [7] and provided in appendix. The external resistance  $R_e$  and inductance  $L_e$  are  $0.19044 \Omega$  and  $0.35364 \text{ mH}$ , respectively as are the ground values  $R_g, L_g$ . The external source connected to the synchronous generator has a voltage of  $v_a(t)=10794\sin(\omega t+1.5172)$  volts. The speed of machine is constant at  $314 \text{ rad/sec}$ . The simulation time is  $0.4 \text{ sec}$ . The winding inductances of this machine are calculated using the equations (14, 18, 20, 21, and 22) depending on the internal fault type. The method was tested by subjecting it to different types of internal faults.

Several internal faults of a phase to ground, two-phases to ground and three-phase to ground were performed in the simulated system. The fault inception time were varied.

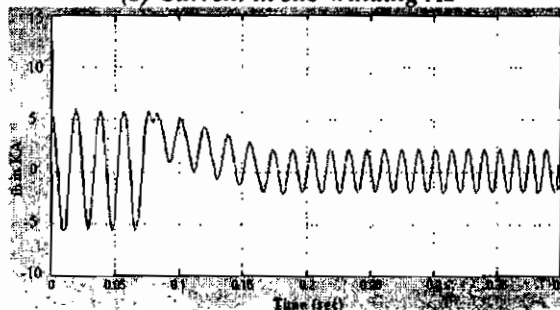
Figure (9) illustrates the stator currents during the single-phase a to ground fault at 75% of winding. The fault inception time is  $0.07 \text{ sec}$ . The currents  $i_{a1}$  and  $i_{a2}$  of windings A1 and A2 are identical prior to the fault. After the fault, the currents in the two resulting subwindings in phase a are in phase opposition (appear in Figs. 9a and 9b) and that very large currents are generated, (since is almost shorted). The currents on both sides of the healthy phases b and c are equal, (i.e.,  $i_{b1} = i_{b2}$  and  $i_{c1} = i_{c2}$ ). Hence, it is sufficient to show only one current for each healthy phase as in Figs. 9c and 9d. The phase voltages (a-g, b-g, and c-g) are also available in Fig. (10). The field current is shown in Fig. (11). The results for phase a to ground are compared with previous approaches [7] that assume sinusoidal winding distributions for the faulted windings. The relative error incurred is very small.



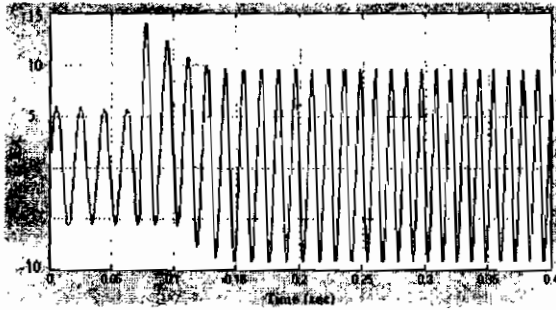
(a) Current in sub-winding A1



(b) Current in sub-winding A2

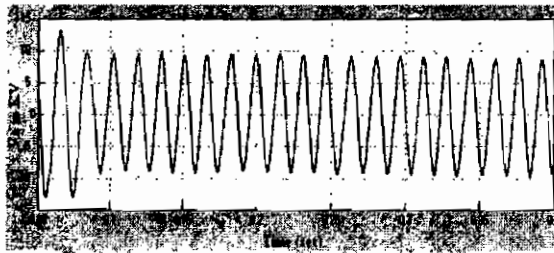


(c) Current in phase b winding

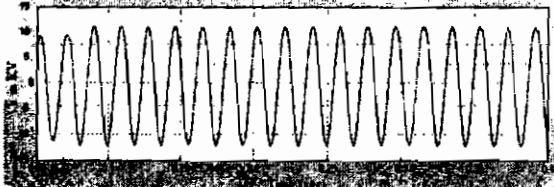


(d) Current in phase c winding

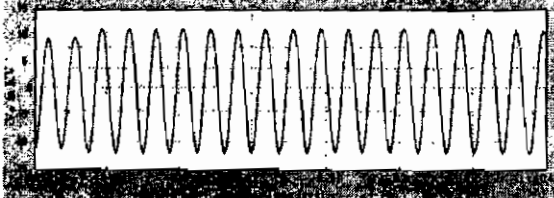
Fig.9 Computed stator currents for an internal phase a to ground fault at 75% of winding.



(a) Voltage across phase a



(b) Voltage across phase b



(c) Voltage across phase c

Fig.10 The three-phase voltage under the condition of phase a to ground fault at 75% of winding.

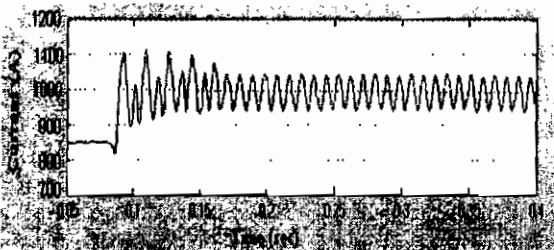
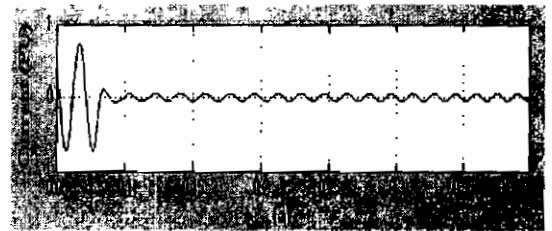


Fig.11 field current under the condition of phase a to ground fault at 75% of winding.

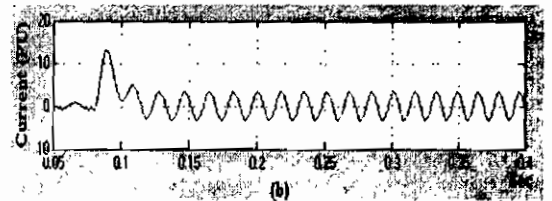
For the internal fault of two-phases to ground, only the current on both sides of the healthy phase c are equal, i.e.  $i_c = i_{c1} = i_{c2}$ . Figs (12), and (13) illustrate the stator

currents and voltages in case of double-phases a and b to ground fault. The fault inception time is 0.08 sec and in 70% of windings a and b.

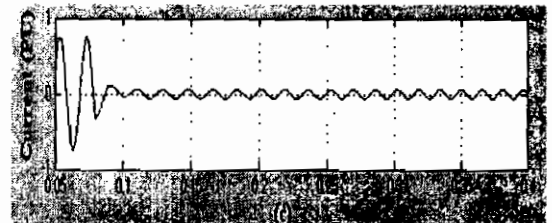
Figure (14) illustrates the stator currents in case of three-phases to ground fault at 0.08 sec and in 60% of windings.



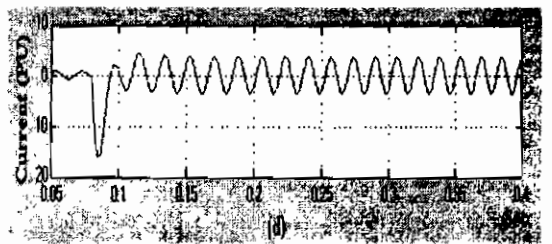
(a) Current in sub-winding A1



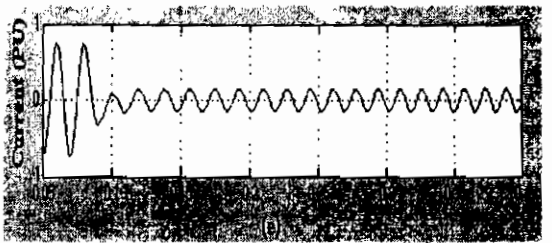
(b) Current in sub-winding A2



(c) Current in sub-winding B1

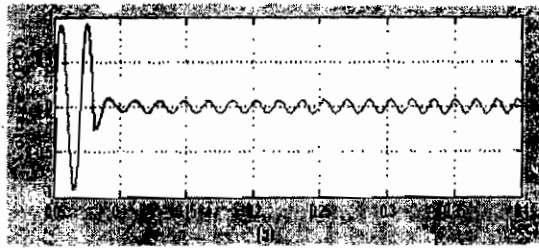


(d) Current in sub-winding B2

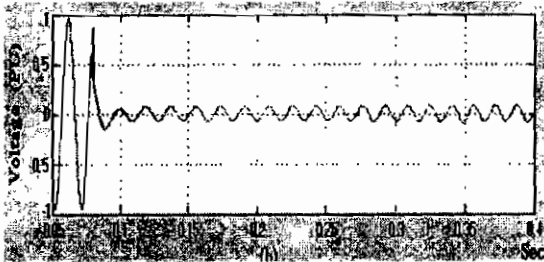


(e) Current in phase C

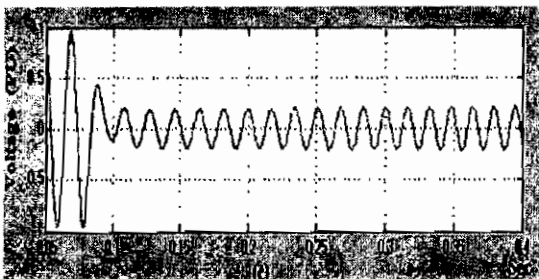
Fig.12 Computed stator currents for an internal phase a to b to ground fault at 70% of winding



(a) Voltage across phase a

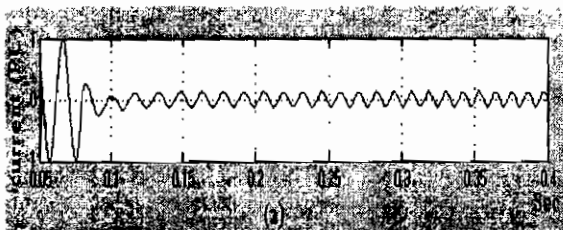


(b) Voltage across phase b

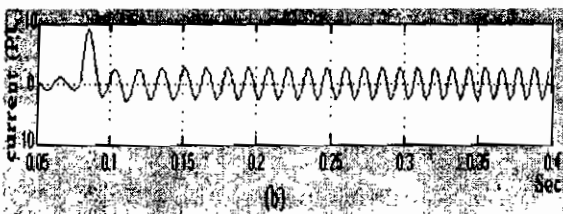


(c) Voltage across phase c

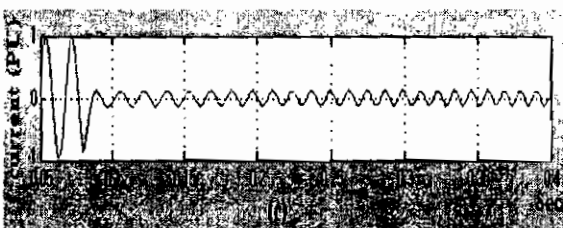
Fig.13 The voltage of three phases under the condition of an internal phase a to b to ground fault at 70% of winding



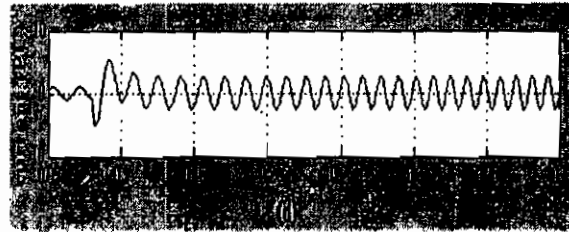
(a) Current in sub-winding A1



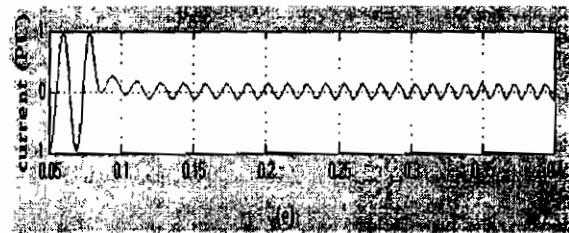
(b) Current in sub-winding A2



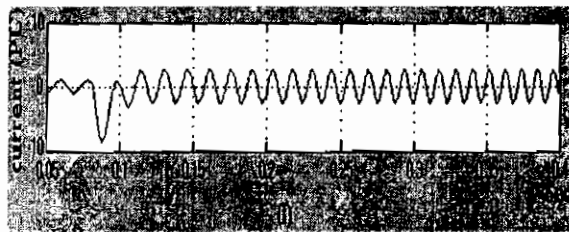
(c) Current in sub-winding B1



(d) Current in sub-winding B2



(e) Current in sub-winding C1



(f) Current in sub-winding C2

Fig.14 Computed stator currents for an internal three-phases to ground fault at 60% of winding

## 6. Conclusion

This paper presents a synchronous machine model in abc form. It calculates the inductances of the machine when the windings are split. The self and mutual inductance of individual windings of the stator are calculated with the aid of special forms. These inductances are calculated using the electrical parameters instead of using the machine geometrical parameters which are difficult to obtain.

The proposed method is based on considering a sinusoidally distribution of MMF and it can take into account the actual shape of the rotor and air gap in inductance calculation. The obtained results are compared with previous approaches [7] that assume sinusoidal winding distributions for the faulted windings. The model has the advantage of simulating any kind of internal faults on any type of windings configuration.

The mathematical model is implemented in MATLAB/SIMULINK. The accuracy and the simplicity make this model more reliable and convenient for the study of internal fault of synchronous generator.

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## 8. Appendices

### Appendix A

Parameters of synchronous machine [7].

Rating 100 MVA  
 Frequency 50 Hz  
 $I_{rag}$  313.2 A  
 Line voltage 13.8 kV

*Table A.1 Generator data*

Manufacturer's parameters	Data
$X_d$	2.03900 (p.u.)
$X_q$	1.94400 (p.u.)
$X_1$	0.12800 (p.u.)
$X_o$	0.09600 (p.u.)
$X_2$	0.14850 (p.u.)
$X_d'$	0.21700 (p.u.)
$X_q'$	0.44600 (p.u.)
$X_d''$	0.15000 (p.u.)
$X_q''$	0.14700 (p.u.)
$R_s$	0.00400 (p.u.)
$T_d$	0.09846 (p.u.)
$T_d'$	0.59757 (Sec)
$T_q'$	0.10485 (Sec)
$T_d''$	0.01521 (Sec)
$T_{do'}$	5.61500 (Sec)
$T_{do''}$	0.02200 (Sec)
$T_{qo'}$	0.45700 (Sec)
$T_{qo''}$	0.04600 (Sec)
$T_q''$	0.01507 (Sec)