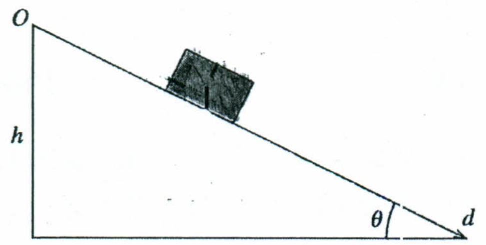
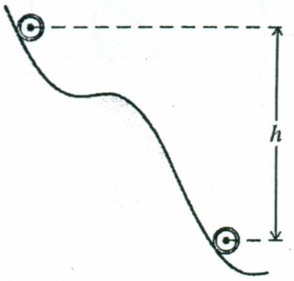
Mansoura University	**Post Graduate**	**15 / 9/ 2013**	**Analytical Mechanics**
Faculty of Engineering	**Math. And Phy. Dep.**		**Time: 3 hours**

Solve as you can. The exam is 11 question in 2 pages.



Prob. 1



Prob. 4

1. A block of mass m is observed accelerating from rest down an incline that has coefficient of friction μ and is at angle θ from the horizontal. (a) How far will it travel in time t? use Newton second law ($x(t) = ?$). ,(b) Use principle of work and energy to find its speed v when it reaches the bottom of the slope, a distance d from its starting point O.

2. A ball is thrown with initial speed v_0 up an inclined plane. The plane is inclined at an angle ϕ above the horizontal, and the ball's initial velocity is at an angle θ above the plane. Choose axes with x measured up the slope, y normal to the slope. (a) Write down Newton's second law using these axes, and (b) find the ball's position as a function of time. (c) Show that the ball lands a distance $R = 2v_0^2 \sin\theta \cos(\theta + \phi)/(g \cos^2\phi)$ from its launch point. Show that for given v_0 and ϕ, the maximum possible range up the inclined plane is:

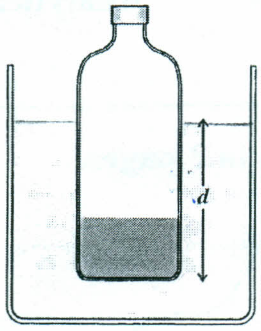
$R_{max} = v_0^2/[g(1 + \sin\phi)]$.

3. A shell traveling with velocity $\mathbf{v}_0$ explodes into three pieces of equal masses. Just after the explosion, one piece has velocity $\mathbf{v}_1 = \mathbf{v}_0$ and the other two have velocities $\mathbf{v}_2$ and $\mathbf{v}_3$ that are equal in magnitude ($v_2 = v_3$) but mutually perpendicular. Find $\mathbf{v}_2$ and $\mathbf{v}_3$ and sketch the three velocities.

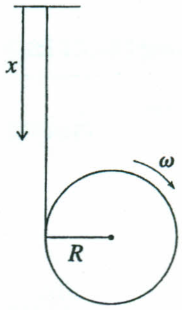
4. A uniform rigid cylinder of radius R rolls without slipping down a sloping track as shown in the figure. Use energy conservation to find its speed v when it reaches a vertical height h below its point of release, where $I = 1/2\, M R^2$.

5. If a particle's potential energy is $U(\mathbf{r}) = k(x^2 + y^2 + z^2)$, where k is a constant, what is the force on the particle?

6. The maximum displacement of a mass oscillating about its equilibrium position is 0.3 m, and its maximum speed is 1.8 m/s. What is the period of its oscillations?



Prob. 7

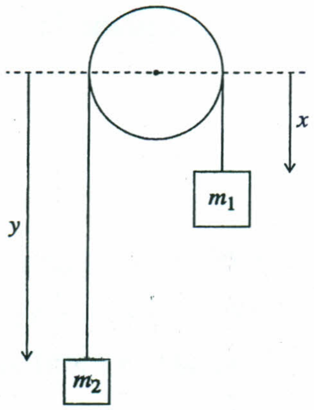


Prob. 9

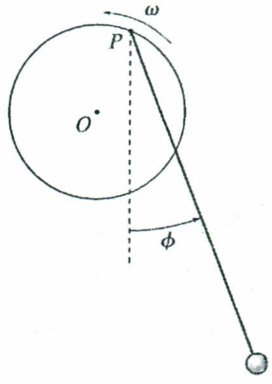
7. A bottle is floating upright in a large bucket of water as shown in the figure. In equilibrium it is submerged to a depth d_o below the surface of the water. Show that if it is pushed down to a depth d and released, it will execute harmonic motion, and find the frequency of its oscillations. If $d_o = 0.4$ m, what is the period of the oscillations?

8. Find the path $y = y(x)$ for which the integral $\left(\int_{x_1}^{x_2} \sqrt{x}\sqrt{1+\acute{y}^2}\,dx\right)$ is stationary.

9. The figure shows a crude model of a yoyo. A massless string is suspended vertically from a fixed point and the other end is wrapped several times around a uniform cylinder of mass m and radius R. When the cylinder is released it moves vertically down, rotating as the string unwinds. Write down the Lagrangian, using the distance x as your generalized coordinate. Find the Lagrange equation of motion and show that the cylinder accelerates downward with $\ddot{x} = 2g/3$.



Prob. 10



Prob. 11

10. Consider the Atwood machine, in which the two masses m_1 and m_2 are suspended by an inextensible string (length l) which passes over a pulley with frictionless bearings, radius R and moment of inertia I. Write down the Lagrangian, using the distance x as generalized coordinate, find the Lagrange equation of motion, and solve it for the acceleration $\ddot{x}$. Compare your results with the Newtonian solution.

11. The figure shows a simple pendulum (mass m, length l) whose point of support P is attached to the edge of a wheel (center O, radius R) that is forced to rotate at a fixed angular velocity ω. At $t = 0$, the point P is level with O on the right. Write down the Lagrangian and find the equation of motion for the angle ϕ.

مع أطيب الأماني بالسعادة .

أ.د./ حامد عوض