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GEOMETRICALLY NONLINEAR ELASTO-PLASIC COLLAPSE AND BUCKLING OF PLATES WITH HOLES

التحليل الهندسي غير الخطي والتصدع المرن ــ اللدن وانبعاج الألواح ذات الفتحات

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ملخص البحث

في هذا البحث تم دراسة التحليل الهندسي غير الخطي والتصدع المرن ــ اللدن للألواح ذات الفتحات كما تم دراسة الانبعاج للألواح ذات الفتحات المستديرة والمربعة.وقد تم استخدام نظرية العناصر الدقيقة المحددة في هذه الدراسة حيث تم اختيار نماذج للألواح باستخدام العناصر المثلثية الشكل وفي هذه النماذج تم تحليل الألواح ذات الفتحات المستديرة والمربعة الموجودة عند المركز حيث تم التأثير بقوى ضغط محوريه عند أحد حواف الألواح المتقابلة .
هذا وقد تم استنتاج حمل الانبعاج للألواح ذات الفتحات المركزية والمعرضة لقوى ضغط محورية عند أحد الحواف المتقابلة للوح . وقد تبين أن حمل الانبعاج لا يتأثر كثيرا بأبعاد الفتحة إلي أن تصبح نصف عرض اللوح بل وربما يتزايد في حالة الأبعاد الأكبر بينما يتأثر الحمل الأقصى لهذه الألواح بشكل كبير بأبعاد الفتحة . وقد تبين أن النقص في قيمة الحمل الأقصى يكون غالبا ملموسا في حالة القيم المنخفضة لنسبة عرض اللوح إلي سمكة .

ABSTRACT

The geometrically nonlinear elasto-plastic aualysis and buckling of perforated plates by the finite element method is described Triangular elements are used to model the plates and a number of solution refinements is discussed The elasto plastic stress strain relationships are based on Ilyushin's approximate area yield function. Solutions are presented for axially compressed square plates with central square and circular holes.

INTRODUCTION

It is often necessary to provide openings in thin plated structures such as cold formed steel members, aeroplane fuselages, plate and box girders and ship structures for accesses and services. The presence of holes in such structures results in a redistribution of the membrane stresses accompanied by a change in the buckling and strength characteristics.

The buckling of perforated plates subjected to pure shear and uniaxial and biaxial compression has been investigated by Rockey et al [1], Pennington-wann [2] and Shanmugam and Narayanan [3] using the finite element method For pure shear, the buckling load decreases continuously with the increasing size of the hole. However, for un-axial and biaxial compression, the buckling load may increase with the increasing size of the hole due to the redistribution of the membrane stresses towards the edges of the plate.

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Even though the buckling load may increase with the increasing size of the hole, it is not to be expected that there will be a corresponding increase in the ultimate or collapse load. This paper describes the buckling and geometrically nonlinear elasto-plastic analysis of perforated plates by the finite element method. Results are presented for uniaxially compressed square plates with central circular and square holes.

Finite element formulations for buckling and geometrically nonlinear elasto-plastic analysis have been presented elsewhere and only a brief outline of the theory is presented herein: Zienkiewicz [4], Kapur and Hartz [5], Roberts and Ashwell [6] and Crisfield [7].

1- ENERGY PRINCIPLES

The total potential energy V of a structural system can be defined by the equation

$$V = V_o - \int P_i . dq_i + \int \left\{ \int \sigma_i d\varepsilon_i \right\} dVol \qquad (1)$$

in which,

V_o is the potential energy of the system prior to the application of external forces,

P_i , q_i represent the external forces and corresponding displacement respectively,

d dimension of hole, and

σ_i , ε_i represent the internal stresses and corresponding strains.

Along any equilibrium path, V is constant, and hence the first and second variations of V along the equilibrium path, denoted by δV and $\delta^2 V$, are zero. Form equation (1),

$$\delta V = P_i \delta q_i + \int \sigma_i \ \ \delta\varepsilon_i dVol = 0 \qquad (2)$$

$$\delta^2 V = -P_i \delta^2 q_i - \tfrac{1}{2} \delta P \delta_i q_i + \int \left(\sigma_i \delta^2 \varepsilon_i + \tfrac{1}{2} \delta\sigma_i \delta\varepsilon_i \right) dVol = 0 \qquad (3)$$

Rearranging equation. (3) gives;

$$\delta P_i \delta q_i = -2 \, P_i \delta^2 q_i + \int \left(2\sigma_i \delta^2 \varepsilon_i + \delta\sigma_i \delta\varepsilon_i \right) dVol \qquad (4)$$

Equation (4) provides the basis for incremental analysis of nonlinear problems. The right hand side of equation (4) is $2\delta^2 V_p$,where $\delta^2 V_p$ being the second variation of V for stationary values of the external forces assuming $\delta p_i = 0$ When $\delta^2 V_p = 0$, equation (4) is indeterminate and hence the vanishing of $\delta^2 V_p$ indicates critical conditions on an equilibrium path Critical conditions occur therefore when;

$$\delta^2 V_p = -P_i \delta^2 q_i + \int \left(\sigma_i \delta^2 \varepsilon_i + \tfrac{1}{2} \delta\sigma_i \delta\varepsilon_i \right) dVol = 0 \qquad (5)$$

2. NONLINEAR STRAINS

An element of a thin plate of thickness t , Young's modulus E and Poisson's ratio ν is shown in Fig. (1). The displacements in the x, y and z directions are denoted by u, v and w and the plate is assumed to have a small initial imperfection w_0. The nonlinear expressions for the membrane and bending strains are: Timoshenko [8]

$$\{\varepsilon_m\} = \begin{bmatrix} \varepsilon_{xm} \\ \varepsilon_{ym} \\ \gamma_{xym} \end{bmatrix} = \begin{bmatrix} u_x + 0.5\left(w_x^2 - w_{ox}^2\right) \\ v_y + 0.5\left(w_x^2 - w_{oy}^2\right) \\ u_y + v_x + w_x w_y - w_{ox} w_{oy} \end{bmatrix} \quad (6)$$

$$\{\varepsilon_b\} = \begin{bmatrix} \varepsilon_{xb} \\ \varepsilon_{yb} \\ \gamma_{xyb} \end{bmatrix} = \begin{bmatrix} -z(w_{xx} - w_{oxx}) \\ -z(w_{yy} - w_{oyy}) \\ -2z(w_{xy} - w_{oxy}) \end{bmatrix} = \begin{bmatrix} -z(\chi_x - \chi_{ox}) \\ -z(\chi_y - \chi_{oy}) \\ -2(\chi_{xy} - \chi_{oxy}) \end{bmatrix} = -z\{\chi - \chi_0\} \quad (7)$$

in which , ε_m is the nonlinear compressive membrane strain,

ε_b is the expression for nonlinear bending strain,

$\gamma_{xym}, \gamma_{xyb}$ are the membrane and bending shear strains,

w, w_0 are the displacement and the initial imperfection in the plate,

and suffices x, y etc. associated with u, v and w denote differentiation

STRESS STRAIN RELATIONSHIPS

For an elastic material, the membrane forces per unit length, N_x, N_y etc. and bending stress resultants M_x, M_y etc. , shown in Fig 1 are related to the strains by the equations

$$\{N\} = \left[N_x, N_y, N_{xy}\right]^T = t[E]\{\varepsilon_m\} \quad (8)$$

$$\{M\} = \left[M_x, M_y, M_{xy}\right]^T = t^3/12[E]\{\chi - \chi_o\} \quad (9)$$

in which,

$$[E] = \frac{E}{\left(1-\nu^2\right)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \quad (10)$$

where:

N,M are the membrane and bending stress resultant respectively .

E is Young's modulus, and

ν is Poisson's ratio

The derivation of the elasto-plastic stress strain relationships, using an approximate area yield function proposed by Ilyushin, was presented by Crisfield [7]. The approximate yield function proposed by Ilyshin is of the form

$$f = \frac{\overline{N}}{t^2\sigma_0^2} + \frac{4s\ \overline{MN}}{\sqrt{3}t^3\sigma_0^2} + \frac{16\overline{M}}{t^4\sigma_0^2} \le 1 \quad (11)$$

in which ; σ_0 is the uniaxial yield stress,

$s = \overline{MN}/\left|\overline{MN}\right|$ and

$$\overline{N} = N_x^2 + N_y^2 - N_x N_y + 3N_{xy}^2$$
$$\overline{M} = M_x^2 + M_y^2 - M_x M_y + 3M_{xy}^2 \qquad (12)$$
$$\overline{MN} = M_x N_x + M_y N_y - 0.5\left(M_x N_y + M_y N_x\right) + 3M_{xy} N_{xy}$$

During plastic flow, the state of stress remains on the yield surface and hence ;

$$\delta f = \{\partial f / \partial N\}^T \{\delta N\} + \{\partial f / \partial M\}^T \{\delta M\} = 0 \qquad (13)$$

Assuming that equation (11) serves also as a plastic potential function, the plastic strain increments can be expressed as :

$$\left\{\delta\varepsilon_m^p\right\} = \lambda\{\partial f / \partial N\} \quad ; \quad \left\{\chi^p\right\} = \lambda\{\partial f / \partial M\} \qquad (14)$$

in which; λ is a positive scalar which defines the absolute magnitude of the plastic strain increments

The total strain increments are the sum of elastic and plastic components , hence :

$$\{\delta\varepsilon_m\} = \left\{\delta\varepsilon_m^e\right\} + \left\{\delta\varepsilon_m^p\right\}$$
$$\{\delta N\} = t[E]\left\{\delta\varepsilon_m^e\right\} = t[E]\left\{\{\delta\varepsilon_m\} - \left\{\delta\varepsilon_m^p\right\}\right\} \qquad (15)$$
$$\{\delta M\} = \tfrac{t^3}{12}[E]\left\{\delta\chi^e\right\} = t^3/12[E]\left\{\{\delta\chi\} - \left\{\delta\chi^p\right\}\right\}$$

Solving equations (11) to (15) gives the elasto-plastic stress strain relationships in the form

$$\{\delta N\} = [CC]\{\delta\varepsilon_m\} + [CD]\{\delta\chi\}$$
$$\{\delta M\} = [CD]^T\{\delta\varepsilon_m\} + [DD]\{\delta\chi\} \qquad (16)$$

4. *FINITE ELEMENT ANALYSIS*

Following well known finite element techniques and assuming the material remains elastic, equation (5) can be reduced to the form

$$\delta^2 V_p = \frac{1}{2}\{\delta q\}^T\left[[KL] + \mu[KG]\right]\{\delta q\} = 0 \qquad (17)$$

in which;

[KL] is the linear stiffness matrix,

[KG] the geometric stiffness matrix which depends on the state of stress prior to buckling,

μ is a scalar load factor, and

{δq} are the nodal displacement variables, which are linear functions,

and hence the term $P_i\delta^2 q_i$ vanishes Critical conditions occur when DET[[KL] + μ [KG]]=0, the lowest eigenvalue μ defining the critical load factor and the corresponding eigenvector the buckled shape.

Using the elastic or elasto- plastic stress strain relationship as appropriate, equation (4) can be reduced to the form .

$$\{\delta P\} = \left[[KEP] + [KGEP]\right]\{\delta q\} \qquad (18)$$

in which; [KEP] depends on the material elastic constants and current state of stress and [KGEP] depends also on the current geometry.

Solutions of nonlinear problems were obtained by incrementing displacements and using a mid increment stiffness teehnique discussed by Roberts and Ashwell [6]. Current states of stress were reduced proportionately back to the yield surface to prevent the build up of errors.

The plates analyzed were modeled using triangular elements Bending action was represented by nine degrees of freedom elements, the displacement function being defined in terms of area coordinates Zienkiewicz [4].

Membrane action was represented by six degrees of freedom elements, linear polynomials being used to define both u and v. In deriving the geometric matrices [KG] and [KGEP] in equations (17) and (18), linear polynomials were used for u,v and w to ensure constant membrane strains throughout an element, which is advantageous for convergence

5- *RESULTS*

The incremental finite element program used in the analysis was developed by Grayson [9]. This program was tested by solving the problem of a simply supported rectangular plate subjected to uniaxial compression. The loaded edges only are being constrained to remain straight The plate was assumed to have an initial imperfection of the form :

$$w_0 = w_{0c} \sin(\tfrac{\pi x}{a}) \sin(\tfrac{\pi y}{b}) \qquad (19)$$

in which:

a, b are the dimensions of plate, and

w_0 , w_{0c} are the initial imperfections

The number of elements used to model the plate was 288 (12 x12 rectangular mesh) and the results were obtained by incrementing the in-plane displacement of the load The results and other details of the problem are shown in Fig 2 and there is an excellent agreement with existing analytical and finite element solutions carried out by Crisfield [7] $\overline{\sigma_m}$ and $\overline{\varepsilon_m}$ are the average compressive membrane stress and strain respectively and ε_0 is the yield strain

The main problem studied was that of a square plate with side length b, containing a central circular or square hole of diameter or side length d, and subjected to uniaxial compression. The loaded edges only were constrained to remain straight in the plane of the plate For displacements normal to the plane of the plate, simply supported and clamped boundary conditions were considered. Values of E, v and σ_0 were taken as 205000 N/mm^2, 0 3 and 245 N/mm^2 respectively. For simply supported plates, w_0 was assumed as given by equation (19) with a = b while for clamped plates w_0 was assumed to be of the form :

$$w_o = w_{oc}\left(1-\cos 2\pi x / b\right)\left(1-\cos 2\pi y / b\right)/4 \qquad (20)$$

The initial central deflection w_{0c} was taken as :

$$w_{oc} = 0.145b \ \left(\sigma_0 / E\right)^{0.5}$$

which has been recommended in the proposed British Code of Practice for the Design of Steel Bridges

A number of radial and rectangular meshes were used to test convergence for the eigenvalue solution. When convergence was satisfactory, the same mesh was used for the corresponding nonlinear problems The critical compressive membrane stress σ_{cr} for initially flat square plates can be expressed as : Timoshenko [8]

$$\sigma_{cr} = K \pi^2 t^2 E / 12\left(1 - \nu^2\right) b^2 \qquad (21)$$

In which K_i is a dimensionless buckling coefficient. K values for perforated plates, obtained from the present analysis arc shown in Fig.(3) and there is general agreement with existing results . Pennington-Wann [2] and Shanmugam and Narayanan [3].

The corresponding results for thc elasto-plastic analysis are shown in Fig.(4) in which $\overline{\sigma}_u$ is the average compressive membrane stress at failurc. There is no significant difference in the results for square and circular holes. To allow for variations in E and σ_0 the results can be normalized in accordance with von Karman's formula for the ultimate strength of uni-axially compressed plates,

$$\overline{\sigma}_u = (\sigma_{cr}\sigma_o)^{0.5} \tag{22}$$

by replacing b/t by (b/t)* where

$$(b/t)^* = (b/t)(205000\ \sigma_0 / 245E)^{0.5} \tag{23}$$

CONCLUSIONS

The buckling load of a uni-axially compressed plate with a centrally placed hole is almost independent of the hole size up to half the width of the plate and may even increase for larger hole sizes

The ultimate load of a uni-axially compressed plate with a centrally placed hole is influenced significantly by the size of the hole. The reduction in the ultimate load is most pronounced for lower b/t values

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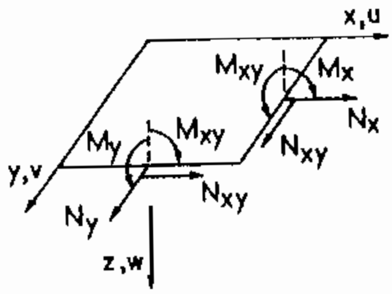

Fig.(1) Element of a plate

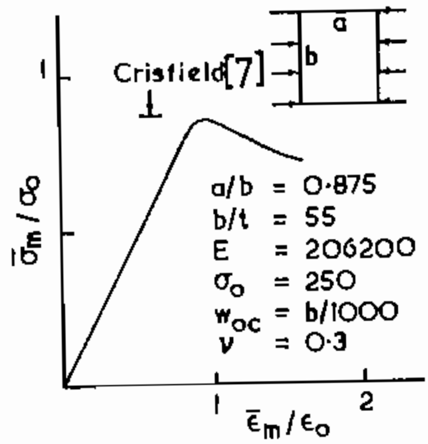

Fig.(2) Collapse of axially compressed plate

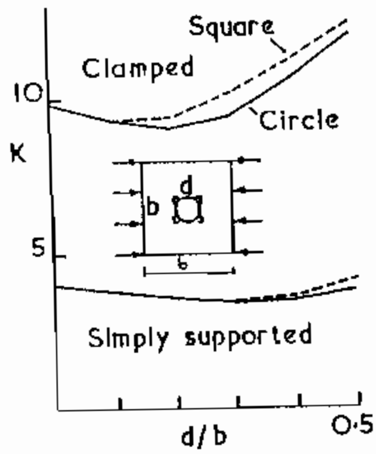

Fig.(3) Buckling of perforated plates

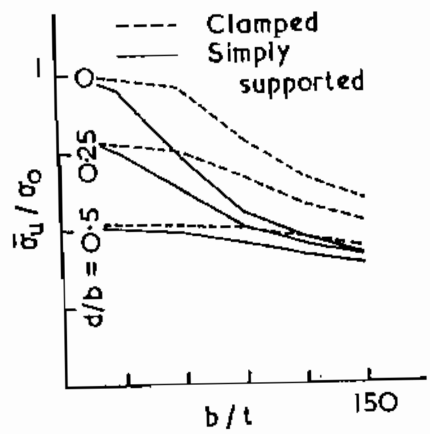

Fig.(4) Failure of perforated plates