A MODEL FOR THE ANALYSIS OF THE TURBULENT FLOW IN A FINITE CIRCULAR GAP

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ABSTRACT :

An analysis of the turbulent flow in a finite circular gap with a moving wall is presented in this paper. According to Ng-Pan's theory,the linearized equations of motion of turbulent flow were numerically integrated based on the apparent kinematic viscosity method for the evaluation of the turbulent kinematic viscosity, and hence a simple isothermal model neglecting the inertia forces is developed for the study of this particular type of flow.

Computation methods were developed for the numerical evaluation of the velocity distribution and consequently the volumetric flow rate, the shear stress calculations were considered for the evaluation of friction losses. The computation methods are directly applied to the study of the hydrostatic supports and thrust bearing in turbo-machines.

INTRODUCTION

The analysis of turbulent flow in narrow gaps has motivated many investigators due to its direct application in the hydrodynamic bearings and the hydrostatic supports. The particular problem of the flow in a circular gap is important for it's practical applications in hydrodynamic thrust bearings and hydrostatic support elements.

In many hydrodynamic bearings laminar flow prevails, but when large Reynolds number exists turbulence can appear. Generally a hydrodynamic bearing operating in turbulent regime has high load carrying capacity but with associated problems of turbulence as energy dissipation and high friction losses. The complete understanding of the turbulent lubrication theory needs the clarification of many aspects of this particular turbulent flow as thermal effects [1], inertia forces [2], .

In the present investigation the principal goal is to present a simplified model to the analysis of this flow neglecting the influence of inertia forces and considering isothermal flow. After [1], it seems that the thermal effect is more pronounced in laminar flow than turbulent flow, which justifies the simplified hypothesis of neglecting thermal effects.

The inertia forces in accelerating flows is responsible of the partial conversion of static pressure energy into dynamic pressure energy. Since this study is devoted to the analysis of this particular flow for practical application, in this stage the influence of inertia forces is omitted. The linearized equations of motion are considered after [3], and [4], the Reichardt's formula of apparent kinematic viscosity is used to evaluate the turbulent kinematic viscosity. The method is out lined in reference [5], but for one dimensional flow. The velocity distribution is predicted and consequently the volumetric flowrate is evaluated. The shear stress can also be evaluated, which enables the calculation of frictional losses.

COVERNING EQUATIONS

The governing equations are the continuity, and momentum equations, following are the major assumptions in treating the equations :

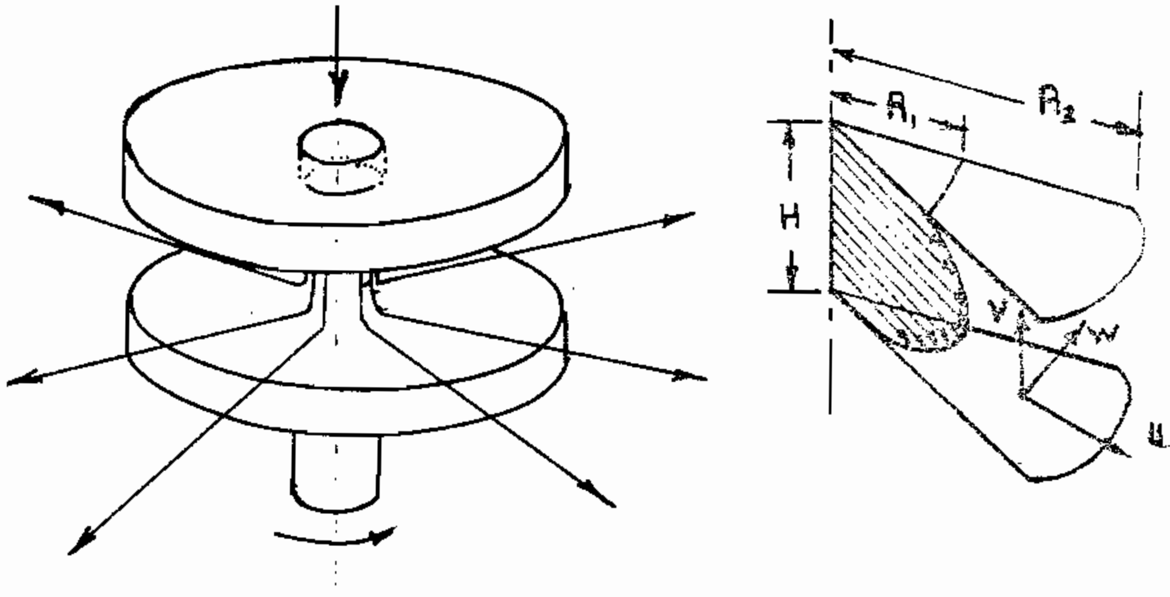

Figure 1 The geometry of the flow.

- Fully devolped steady isentropic, isothermic turbulent flow.
- Incompressible , Newtonian fluid.
- Externally pressurised, $\frac{\partial P}{\partial z} = o$ also $\frac{\partial P}{\partial \theta} = o$
- The gap height is much smaller than the raduis R ($R \gg H$) .
- Velocity gradients across the film thickness are lager than all other velocity gradients .
- No thermal or mechenical distortion accurs .
- Body forces, inertia forces are negligeable .
- Isentropic eddy-diffusivity .
- Local wall shear stress is a function of the local mean flow Reynolds number and does not depend on the flow details associated with pressure gradients and the converging flow channel .

The general form of the continuity equation is as follows :

$$\frac{1}{R} \frac{\partial}{\partial R} (RU) + \frac{1}{R} \frac{\partial V}{\partial \theta} + \frac{\partial W}{\partial Z} = 0$$

For a finite gap and based on the above mentioned assumptions it reduces to the following form :

$$\frac{\partial RU}{\partial R} + \frac{\partial V}{\partial \theta} = 0 \quad \ldots(1)$$

Based on the above mentioned assumptions the momentum equation is written on the following forms :

$$-\frac{\partial P}{\partial R} = \frac{\partial \tau_R}{\partial Z} = \mu \frac{\partial^2 U}{\partial Z^2} - \frac{\partial}{\partial Z}(p.\overline{u'.v'}) \quad \ldots(2)$$

$$-\frac{1}{R}\frac{\partial V}{\partial R} = \mu \frac{\partial^2 V}{\partial Z^2} - \frac{\partial}{\partial Z}(p.\ v) \quad \ldots(3)$$

with $\frac{\partial P}{\partial \theta} = 0$ and consequently $\frac{\partial \tau_\theta}{\partial Z} = 0$

$$-\frac{\partial P}{\partial Z} = 0$$

The boundary conditions of these equations are :

$$U(R, \theta, Z) \begin{cases} Z = 0 \\ Z = H \end{cases} \quad U = 0 \quad \ldots(1)$$

$$V(R, \theta, Z) \begin{cases} Z = 0 & V = \omega R \\ Z = H & V = 0 \end{cases} \quad \ldots(2)$$

$$w(R, \theta, Z) \begin{cases} Z = 0 \\ Z = H \end{cases} \quad W = 0 \quad \ldots(3)$$

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- MODEL FOR THE VELOCITY DISTRIBUTION :

The Boussinesqu's eddy-diffusivity formula is as follows :

$$-\overline{u'v'} = \epsilon . \frac{\partial U}{\partial Z}$$

Introducing Boussinesqu's formula in equation (2) becomes :

$$\frac{\partial P}{\partial R} = \mu \frac{\partial}{\partial Z}\left(1 + \frac{\epsilon}{\nu}\right)\frac{\partial U}{\partial Z} \quad \ldots(4)$$

Integrating equation (4) with the foregoing boundary conditions (1), bearing in mind that equation (4) is non linear since the diffusivity is a function of Z, thus the equation contains terms which should be numerically determined. The first integration with respect to Z will give :

$$\left(1 + \frac{\epsilon}{\nu}\right) . \frac{\partial U}{Z} = \frac{1}{\ \ } \frac{dP}{dR} Z + C_1 \quad \ldots(5)$$

For $Z = 0$, $C_1 = \left(1 + \frac{\epsilon}{\nu}\right) . \frac{\partial U}{\partial Z}\Big|_{Z=0}$

After Boussinesq's equation the wall shear stress could be written on the following form :

$$\tau_o = \mu \left(1 + \frac{\epsilon}{\nu} \right) \cdot \frac{\partial U}{\partial Z} \Big|_{Z=0} \qquad \ldots (6)$$

Consequently the constant $C_1 = \tau_o / \mu$ ... (7)

Substitute (7) in (5) and integrate with respect to Z

$$U(R,\theta,Z) = \frac{dP}{dR} \int_o^Z \frac{Z \; dZ}{u\left(1+\frac{\epsilon}{\nu}\right)_R} + \tau_o \int_o^Z \frac{dZ}{u\left(1+\frac{\epsilon}{\nu}\right)} \qquad \ldots (8)$$

Put $\left(1 + \frac{\epsilon}{\nu} \right) = F(Z)$, in equation (8) , and after equation (7) , C_1 becomes :

$$C_1 = -\frac{dP}{dR} \cdot \frac{\int_o^H Z \; dZ / \mu F}{\int_o^H dZ/F} \qquad \ldots (9)$$

Then equation (8) , becomes :

$$U(R,\theta,Z) = \frac{dP}{dR} \int_o^Z \frac{Z \; dZ}{\mu F} - \frac{dP}{dR} \frac{\int_o^H Z \, dZ / \mu F}{\int_o^H dZ/F} \int_o^Z \frac{dZ}{F}$$

Or

$$U(R,\theta,Z) = \frac{dP}{dR} \left[\int_o^Z \frac{Z \, dZ}{\mu F} - \frac{\int_o^H Z \, dZ/\mu F}{\int_o^H dZ/F} \cdot \int_o^Z \frac{dZ}{F} \right] \qquad \ldots (10)$$

Now introducing the nondimensional notation :

$Z^+ = Z / H$ $\qquad$ $U^+ = U / \omega R_1$ $\qquad$ $R^+ = R/R_1$

$R = H^2 \omega / \nu = \frac{VH}{\nu} \, (H/R_1)$

$P^+ = P \, (H/R_1)^2 \, \frac{1}{\omega \mu}$

Then the nondimensional velocity distribution is as follows :

$$U^+(R,\theta,Z) = \frac{H^2}{\mu V_1} \frac{dP}{dR} \left[\int_o^{Z^+} \frac{Z^+ \, dZ^+}{F} - \frac{\int^1 Z^+ \, dZ^+/F}{\int^1 dZ^+/F} \int_o^{Z^+} \frac{dZ^+}{F} \right] \qquad \ldots (11)$$

Introducing the dimensionless pressure gradient :

$$P^* = \frac{dP}{dR} \Big/ \frac{\mu V_1}{H^2} \qquad \ldots (12)$$

The pressure gradient coeffient is defined as follows :

$$R_P = \frac{H^3}{\mu \nu} \frac{dP}{dR} \qquad \ldots (13)$$

After equations (12), (13) the dimensionless pressure gradient is written on the following form :

$$P^* = \frac{R_p}{R_e} \cdot \frac{H}{R_1}$$

Put $I\,(Z^+) = \int_o^{Z^+} \frac{d\,Z^+}{F}$, $J\,(Z^+) = \int_o^{Z^+} \frac{Z^+}{F}\, d\,Z^+$ ...(14)

After equations(12), (13), (14), equation (11) could be written on the following form :

$$U^+_{(R,\,\Theta,\,Z)} = P^* \left[J\,(Z^+) - \frac{J\,(1)}{I\,(1)} \quad I\,(Z^+) \right] \qquad \text{...(15)}$$

Equation (15) is the equation of the velocity distribution in nondimensional form. The mean flow velocity over a circular gap is defined as follows :

$$\overline{U}_{(R)} = \frac{1}{H\,2\pi R} \int_o^H U \,.\, 2\pi R \;\; d\,Z$$

Or in nendimensional form becomes :

$$\overline{U}^+_{(R)} = \int_o^1 U^+\,(\,R\,,\,0,\,Z\,)\,d\,Z^+ \qquad \text{...(16)}$$

Assume pure Poiseuille flow, pure Couette $dP/dR = 0$ at $R=R_2$. This to avoid problems arise from the non uniformity of velocity distribution at entrance, thus :

$$\overline{U}^+_{(R_1)} = \int_o^1 P^* \left[J\,(Z^+) - \frac{J\,(1)}{I\,(1)} \,.\, I\,(\,Z^+) \right] dZ^+ \qquad \text{...(17)}$$

Put $\overline{I} = \int I\,(\,Z^+)\;\,d\,Z^+$, $\overline{J} = \int J\,(\,Z^+)\,d\,Z^+$

and $G_r = \overline{J} - \frac{J\,(1)}{I\,(1)}\,\overline{I}$ thus equation (17) becomes :

$$\overline{U}^+_{(R_1)} = P^* \,.\, C_r \qquad \text{...(18)}$$

SHEAR STRESS CALCULATION

Assuming linear distribution of the shear stress and after equations (6), (7) equation (9) can be written on the following form :

$$\frac{\tau_o}{\mu} = -\frac{d\,P}{d\,R} \, \frac{\int_o^H \frac{Z}{\mu}\,\frac{d\,Z}{F}}{\int_o^H \frac{d\,Z}{F}} = \frac{1}{\mu} \cdot \frac{d\,P}{d\,R}\; H \; \frac{\int_o^1 Z^+ \frac{d\,Z^+}{F}}{\int_o^1 \frac{d\,Z^+}{F}} \qquad \text{...(19)}$$

Introducing the function J(1), I(1), after équation (14), in equation (12) :

$$\tau_o = -\frac{d\,P}{d\,R}\; H \; \frac{J\,(\,1\,)}{I\,(\,1\,)} \qquad \text{...(20)}$$

Rearanging equation (20), and introducing the ratio between gap height and inner raduis, $\psi = H/R_1$ to obtain the shear stress equation on the following form :

$$R_e = - \frac{T}{P\gamma} \cdot \psi \cdot \frac{I(1)}{J(1)} \qquad \ldots (21)$$

Where T is the dimensionless shear stress defined as $T_o = \frac{\tau_o H^2}{\mu \nu}$

Equation (21) provides the relation between the Reynolds number and the pressure gradient. For linear distribution of the shear stress one can write the following relation:

$$\tau = \tau_o + \frac{dP}{dR} \cdot Z \qquad \ldots (22)$$

Or in nondimensional form becomes :

$$T_{(Z)} = T_o + R_p Z^+ \qquad \ldots (23)$$

After equation (21) and the definition of the pdimensionless pressure coefficient

$$T_o = - R_p \cdot \frac{J(1)}{I(1)} \qquad \ldots (24)$$

Apply equation (23) on the upper wall where Z = H, stationary wall, and after equation (24), the following expression of the shear stress on the upper wall is obtained :

$$T_u = R_p \left(1 - \frac{J(1)}{I(1)}\right) \qquad \ldots (25)$$

the values of the shear stresses are presented in form of ratios between the shear stresses and the Reynolds number as following :

$$\frac{T_o}{R_e} = \frac{\tau_o}{\mu \frac{V_1}{H} \psi} \qquad \frac{T_H}{R_e} = \frac{\tau}{\mu \frac{V}{H} \psi} \qquad \ldots (26)$$

METHOD OF SOLUTION:

A computer program is written in Fortran code. The program flow chart is shown in figure (2). The inputs are; speeds, shear stress ratios , pressure coefficients, lubrication oil characteristics and the support geometric characteristics. Equations (14) were numerically integrated. The Reichard's formula for the calculation of turbulent apparent kinematic viscosity is used. (equations (27) through (34) The results are introduced in equation (17), then equations (14) and (17) are numerically integrated using Simpsons rule. Finally an area integration is under taken to find the discharge.

A medium lubrication oil characteristics were choosen, the dynamic viscosity μ = 0.0486Kg/ms, kinematic viscosity ν = 0.000054 m2 /s. A cases of negative pressure gradient, and positive pressure gradient were considered. The program is also re-written in Basic code, since most of the small computers use Basic. The program.may be refined in the future in order to provide the required curves and 3-D graphics of the support.

RESULTS AND DISCUSSIONS:

Sample results are presented in figures (3),(4),(5),(6). Two representative cases are considered:

- Positive gradient dp/dx, i.e pressure rise in the positive direction of the radial coordinate.
- Negative pressure dradient dp/dx, i.e pressure drop in the positive direction of the radial coordinate.

Several runs are performed at variable speeds, and shear ratios. Only the velocity distribution in radial direction is presented, since it constitutes the first step in the calculation of the volumetric flow rate. In both cases the velocity distribution is obtained for different Reynolds number values.

Figures (3)and(4) show examples of velocity distribution obtained from this flow theory, for positive and negative pressure gradients. The curves show typical form of turbulent flow; which is characterized by pronounced velocity gradients in the vicinity of the walls. A separation occurs near the upper wall, the magnitude of reverse velocity distribution is decreased with decreasing the Reynolds number.

The volumetric flow in nondimensional form is presented in figure(5) for negative pressure gradient, and figure(6) for positive pressure gradient. This curve is the basis for determining the volumetric flow in a gap between two disks.

CONCLUSI ONS:

The presented model provides a simple rapid method for the analysis of turbulent flow between two disks. It provides a detailed informations of the flow conditions, which are useful for rapid determination of the flow characteristics, as follow:

- Equation(11) which is presented in figures(3) and (4) gives the velocity distribution.
- Equation(18) which is presented in figures(5),(6) gives the volumetric flow rate.
- Equations(24)and(25) allow the determination of shear stress .

Certainly the validity of the assumed hypothesis must be experimentally verified.

NOMENCLATURE:

H	Gap height	(m)
P	Pressure	(N/m^2)
R	Radial coordinate	(m)
U	Flow velocity, radial component	(m/s)
V	Flow velocity, vertical component	(m/s)
W	Flow velocity, 0 component	(m/s)
Z	Vertical coordinate	(m)
ϵ	Apparent kinematic viscosity of turbulent flow	(m^2/s)
μ	Dynamic viscosity of the fluid	(Kg/ms)
ν	Kinematic viscosity of the fluid	(m^2/s)
ρ	Density of the fluid	(Kg/m^3)
τ	Shear stress	(N/m^2)

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REFERENCES

1- Huebner, K.H., "Solution for the Pressure and Temperature in Thrust Bearing Operating in the Thermohydrodynamic Turbulent Regime" J. of Lub. Technology Trans. ASME. January 1974, pp. 58.
2- Constantinescu, V.N., Caletuse, S., "On the Possibilities of Improving the Accuracy of the Evaluation of Inertia Forces in Laminar and Turbulent Films" J.of Lub. Technology Trans. ASME, January 1974, pp. 69.
3- Ng, C.W., and Pan,C.H.T.,"A Linearized Turbulent Lubrication Theory" J. of Basic Engineering, Trans ASMEvol 87 no 3 pp. 675 Sept 1965.
4- Ng,C.W.,"Fluid dynamic Foundation of Turbulent Lubrication Theory" ASLE Trans vol 7 pp. 311 1964.
5- Tognola,S.,"A nalysis of Turbulent Flow in gap with moving wall" Escher Wyss news 1/2 1980.
6- Hinze,J.O., "Turbulence, McGraw Hill, New york 1975.

APPENDIX

The Reichardtś Formula and The Calculation of Turbulent Apparent Kinematic Viscosity

The Reichardtś formula envolves the calculation of the turbulent eddy diffusivity:

$$\frac{\epsilon}{\nu} = k^* \left[\frac{Z}{\nu} \frac{\tau_o}{\rho} - \delta^+ \tanh \left(\frac{Z}{\nu} \frac{1}{\delta^+} \frac{\tau}{\rho} \right) \right] \quad \ldots (27)$$

where k^*, δ^+ are emperical constsnts whose values are :

$$0.3 \leqslant k \leqslant 0.4 \qquad\qquad 5 \leqslant \delta^+ \leqslant 15$$

Introducing nondimensional notations, equation (27) becomes :

$$\frac{\epsilon}{\nu} = k \left[Z^* - \delta^+ \tanh \left(\frac{Z^*}{\delta^+} \right) \right] \quad \ldots (28)$$

With

$$Z^* = \frac{Z}{\nu} \sqrt{\frac{\tau}{\rho}}$$

In solving equation (25), the difficulties arise from the unknown reciprocal influence of two closely spaced walls, the stationary and moving walls. A simplified hypothesis should be introduced to overcome this problem, So that the eddy diffusivity in the vacinity of each wall is influenced only by the corresponding wall and the line of demarcation is Z_m defined by :

$$\frac{Z_m}{\nu} \sqrt{\frac{\tau_w}{\rho}} = \frac{H - Z_m}{\nu} \sqrt{\frac{\tau_w}{\rho}} \quad \ldots (29)$$

As an approximation $Z_m = (1/2)\ H$ thus :

$$Z^* = \frac{Z}{\nu} \sqrt{\frac{\tau}{\rho}} \qquad 0 \leqslant Z \leqslant H/2 \quad \ldots (30)$$

$$Z^* = \frac{H - Z}{\nu} \sqrt{\frac{\tau}{\rho}} \qquad H/2 \leqslant Z \leqslant H \quad \ldots (31)$$

In nondimensienal notation equations (30), (31) becomes :

$$Z^* = Z^+ \sqrt{T} \qquad 0 \leqslant Z^+ \leqslant 1/2 \quad \ldots (32)$$

$$Z^* = (1 - Z^+) \cdot \sqrt{T} \qquad 1/2 \leqslant Z^+ \leqslant 1 \quad \ldots (33)$$

After equations (27) through (33) and using the numerical values of the constants k^*, δ^+ the eddy-diffusivity is written on the following form :

$$\frac{\epsilon}{\nu} = 0.4 \left[Z \sqrt{T} - 10.7 \tanh \left(Z \sqrt{T} / 10.7 \right) \right] \quad \ldots (34)$$

with $Z = Z^+$ for $0 \leqslant Z \leqslant 0.2$

$Z = 1 - Z^+$ for $0.8 \leqslant Z^+ \leqslant 1$

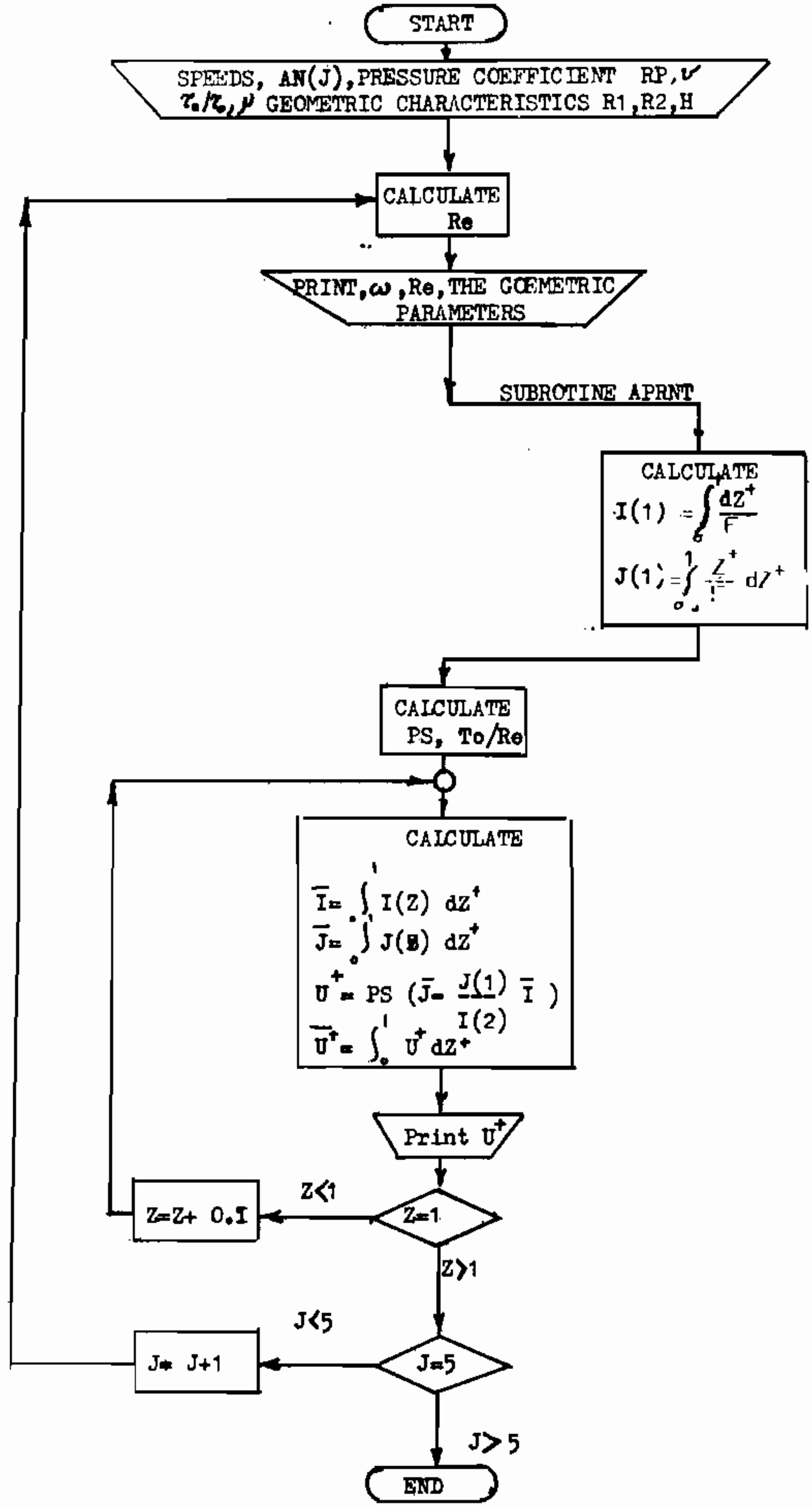

Figure 2. Computer program flow chart.

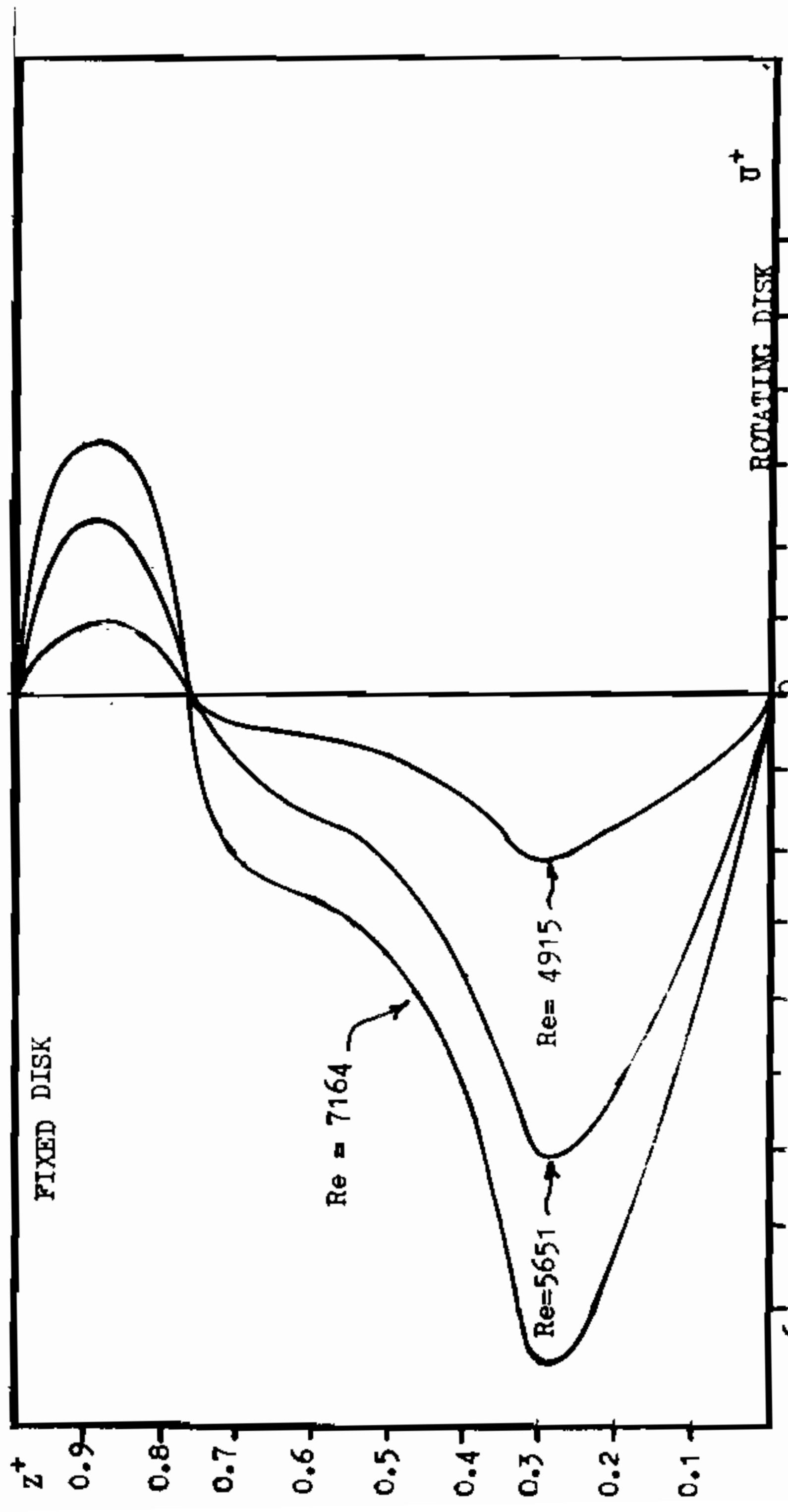

Figure 3. Turbulent velocity distribution in a gap between two disks with positive pressure gradient. ($R_p = 10^5$)

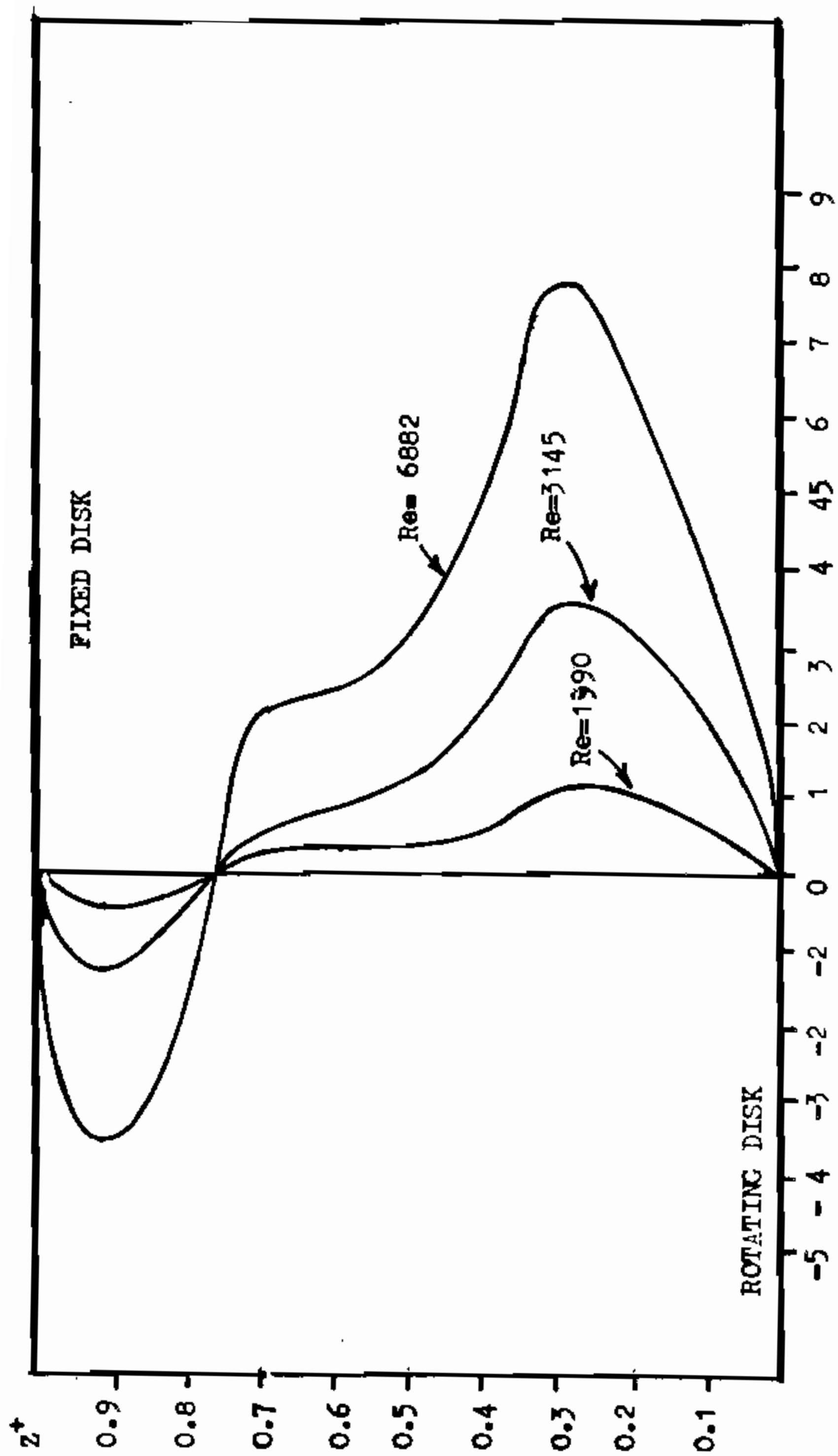

Figure 4. Turbulent velocity distribution in a gap between two disks, with negative pressure gradient.(Rp=- 50x 10^3)

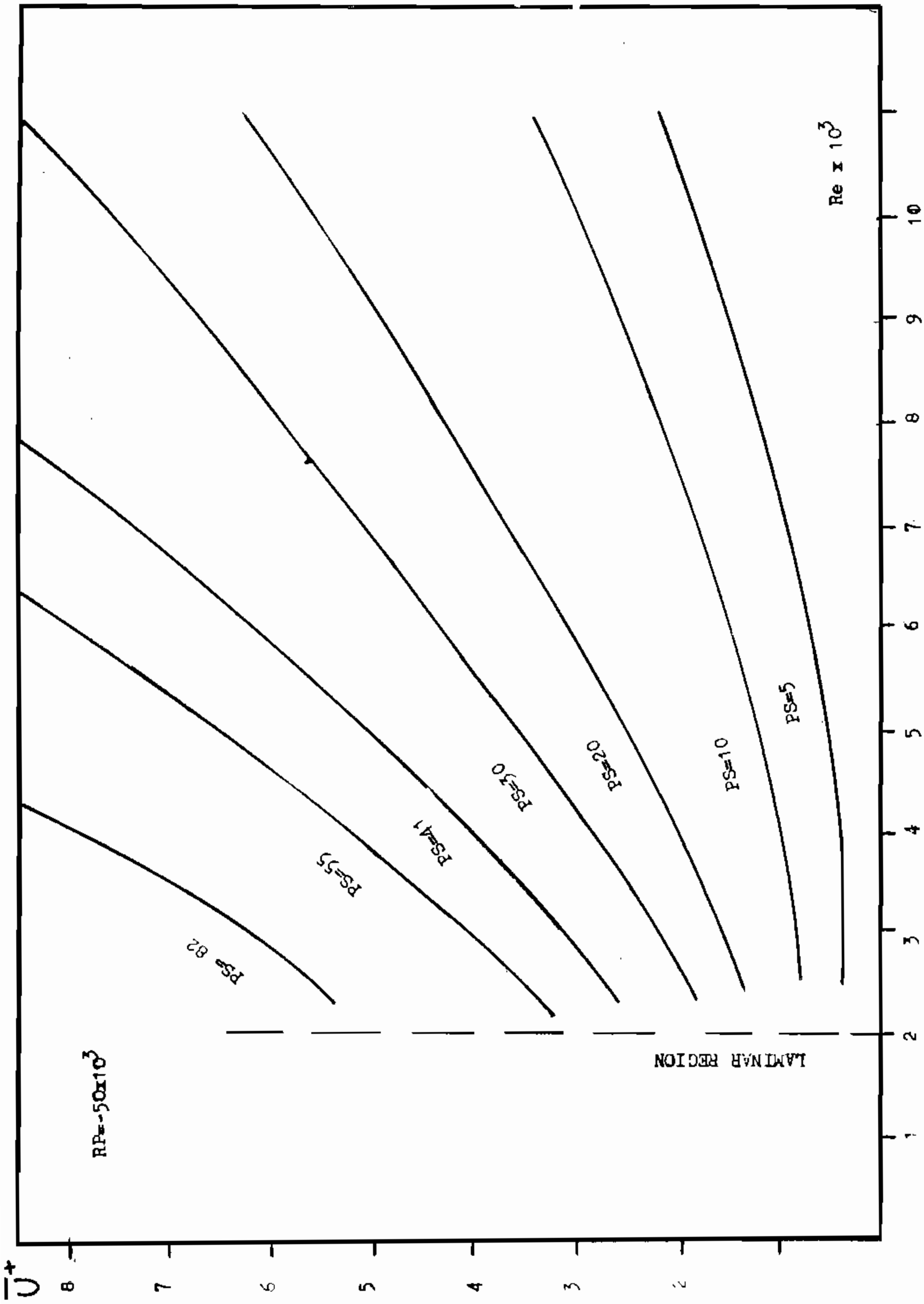

Figure 5. Mean dimensionless flow velocity $\bar{U}^+$ in a gap with negative pressure gradient.

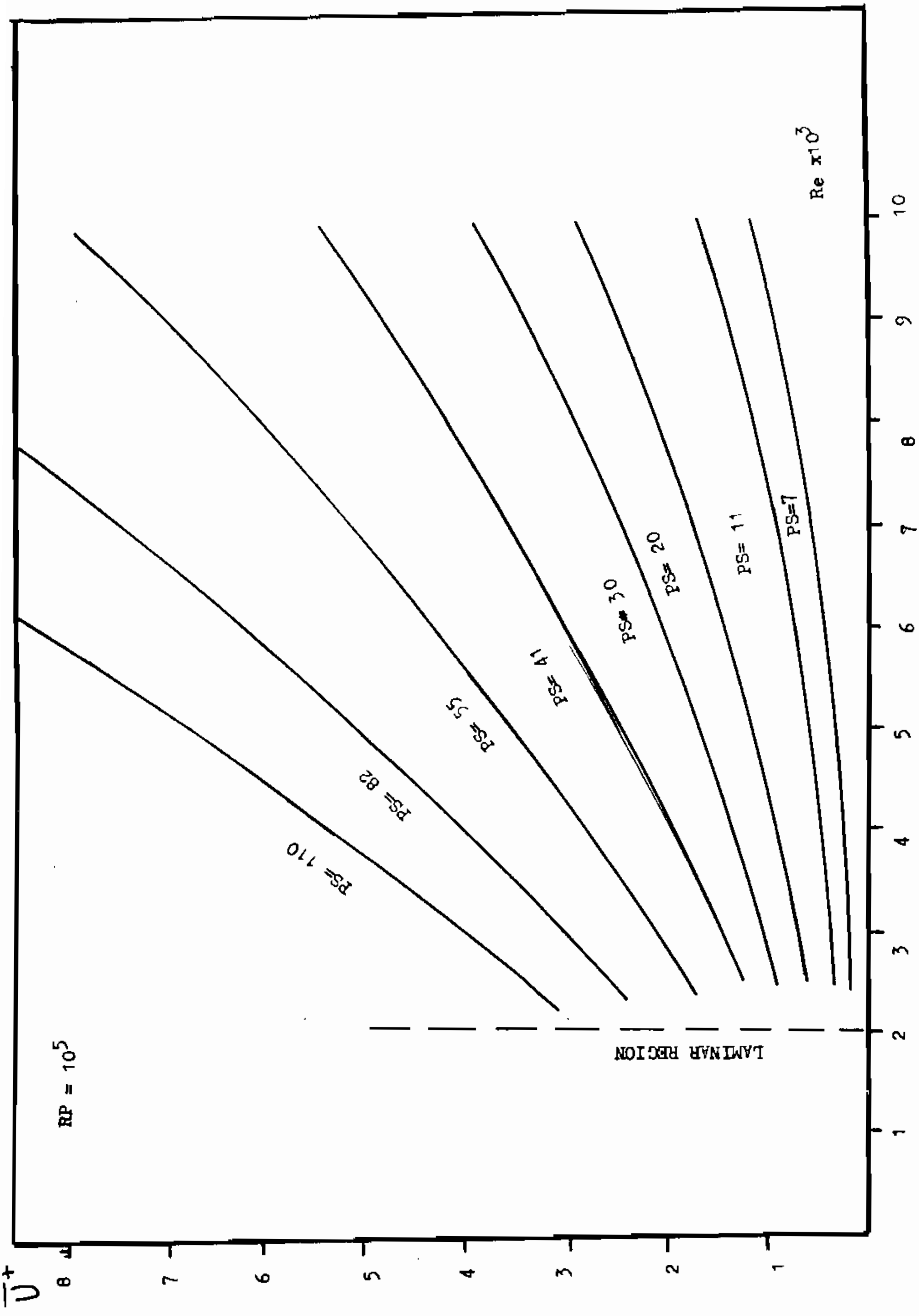

Figure 6. Mean dimensionless flow velocity U^+ in a gap with positive pressure gradient.